RF Noise and Longitudinal Emittance in an Electron Storage Ring

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Dispersions in phase and energy of beam bunches in an electron storage ring are systematically derived by solving difference equations when RF phase noises are significant. An effect of the discrete revolution on the natural emittance is shown. Criteria on the characteristics of RF in the SPring-8 storage ring are obtained.

Noises in RF system in proton storage rings, where phase feedback between beam and RF is essential, cause major problems to be solved to keep long term stability against diffusion [1]. In an electron storage ring, they are negligible in view of stability because the stochastic process of quantum radiation smears out any history of motion. However, to obtain an equilibrium emittance or distribution, RF noises must be included in the same way as quantum excitation. Furthermore, when peaks in the frequency spectrum of RF noises exist, they induce systematic oscillations in the longitudinal motion of beam bunches. These oscillations may affect the equilibrium bunch length significantly when the frequency of noises harmonizes with that of the synchrotron oscillation.

In this paper [2], we solve difference equations for the longitudinal motion of particles in an electron storage ring under the influence of the above mentioned noises. Our assumption is that the RF noises acting as disturbances in bunch centers give the maximum longitudinal dispersions of a bunch. We give the equilibrium rms value of a longitudinal bunch distribution and obtain conditions on the strength of these noises to suppress the spread of the beam below a certain limit. An aim to use the difference equations instead of the conventional continuous equations is to understand effects of discrete revolution on the longitudinal emittance.

The equations of motion for the linearized synchrotron oscillation in a storage ring with an RF gap are given by

$$\begin{cases} \Delta\delta\phi_i = 2\pi\Gamma_s^{-1}\delta E_i \\ \Delta\delta E_i = -2\pi\Gamma_s\dfrac{\Omega_s^2}{\omega_s^2}\Delta\delta\phi_{i+1} - \dfrac{4\pi\alpha_s}{\omega_s}\delta E_i + Q_i \end{cases}$$

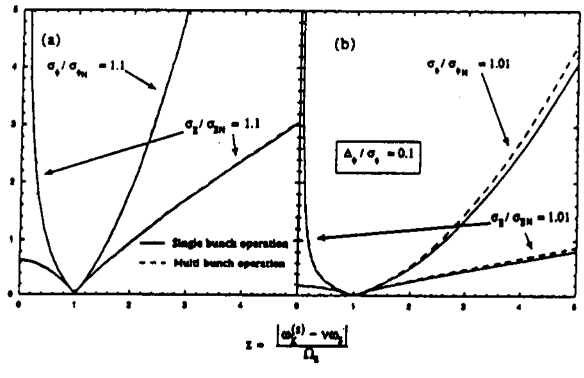
where the variables $\delta\phi_i = \phi_i - \phi_s$ and $\delta E_i = E_i - E_s$ denote deviations from synchronous values of phase with respect to that of RF acceleration voltage and energy, respectively, at the *ith* turn. Δ stands for the change in a turn: $\Delta X_i = X_{i+1} - X_i$ for a variable X. α_s stands for the radiation damping coefficient for the synchronous particle and Q_i denotes energy fluctuation due to quantum radiation in the *ith* turn. Ω_s is the angular frequency of the plain synchrotron oscillation and $\Gamma_s = \beta_s{}^2E_s/h\eta_s$ is a constant with dimension of energy. Values for fundamental machine parameters are listed in Ref. [3]. For the RF noise, we replace the factor $\delta\phi_{i+1}$ in the second line of the E.O.M..by $\delta\phi_{i+1} + \Phi_{i+1}$ with Φ_{i+1} being the noise.

Solving the equations of motion with the noises, we have evaluated dispersions σ_ϕ, and σ_E, for the phase and energy, respectively, at the stationary state of the motion. They are written in a form $\sigma_X^2 = \sigma_{XN}^2\,(\,1 + R_X^{(w)} + R_X^{(s)}\,)$ for $X = \phi$ or E, where σ_{XN}^2 is the natural dispersion squared, namely the contribution from the quantum emission noise, $R_X^{(w)}$ and $R_X^{(s)}$ are the contributions from the white and systematic RF noises, respectively, normalized by σ_{XN}^2. The systematic RF noise is introduced as a component in Φ_{i+1} as one chraracterized by strength S_k and frequency $\omega_k^{(S)}$

We consider a condition that the maximal increase of the equilibrium bunch length or dispersion in bunch centers due to the effect of RF noises must be smaller than a certain limit. We write

$$p > \sigma_X/\sigma_{XN} = \sqrt{1 + R^{(w)} + R_X^{(s)}}\,,$$

for a given value of $p > 1$. Since the r.h.s is a function of S_k, $\omega_k^{(S)}$ and the strength of the white RF noise, Δ_ϕ, we obtain an allowed region in these three parameters for a given p. To describe resonant behaviour of $R_X^{(s)}$, an adequate variable is found to be $z = \dfrac{|\,\omega_k^{(s)} - \nu\omega_s|}{\Omega_s}$, where $\nu = \left[\dfrac{\omega_k^{(s)}}{\omega_s} + \dfrac{1}{2}\right]_G$ and $[\cdots]_G$ denotes the Gauss's symbol. Figures show boundaries of the condition in z-$(S_k/\sigma_{\phi N})$ plane for different values of p. $\Delta_\phi/\sigma_{\phi N}$ is fixed to 0.1.



These figures show the condition puts a strong constraint on S_k in the vicinity of the resonance point, $z = 1$. They also show that the condition for the phase gives stronger constraint in $z < 1$ compared with that for the energy. This is a result of considering a constant phase shift at the RF acceleration ($z = 0$) and taking an ensemble of different modules of the shift.

To measure the effect of the discrete revolution on the natural energy spread, we consider a quantity

$r_E = [(\sigma^2_{EN})_{Discrete} - (\sigma^2_{EN})_{Continuous}] / (\sigma^2_{EN})_{Discrete}$

where meaning of suffices is apparent and $(\sigma^2_{EN})_{Continuous}$ is given in Ref. [4]. r_E is found to be 0.002 in the SPring-8 case while it takes a value 0.13 (0.05) for the case of Tristan (TristanII]. The small value of r_E for the SPring-8 case is a result of small synchrotron tune.

References

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