On Radiation Spectrum from a Quasi-periodic Undulator

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Ordinary undulators consist of a periodic array of magnet poles of alternating polarities. The radiation emitted in each magnet pole interferes with each other, producing enhanced emission at a fundamental frequency and its harmonics. Since a mixture of the harmonics degrades the ratio of signal to noise in many experiments, the higher harmonics are required to be eliminated. In the high energy region of x-rays above 30 keV, however, it is practically difficult to exclusively pick up the fundamental radiation. Hashimoto and Sasaki [1] proposed a new undulator, which comprises a quasi-periodic array of magnet poles and will be called "Quasi-periodic undulator" (hereafter, referred to as QPU). No rational higher harmonics of the fundamental frequency are contained in the radiation from the QPU. Here we give a breaf explanation of the radiation spectrum from a quasi-periodic undulator.

In a normal planer periodic undulator (PU) an electron moves sinusoidally in the horizontal plane. It takes the time $\lambda_0/c\bar{\beta}_z$ or the electron to traverse one period of the undulator, where $\bar{\beta}_z$ is the average longitudinal velocity. During this time the light travels the distance $L=\lambda_0/\bar{\beta}_z$, so that the light emitted by an electron at a top of the sinusoidal motion precedes the one emitted at the next top by $\Delta\phi=2\pi\omega/\omega_1$ in the phase. Here is the resonant frequency of the undulator and given by $\omega_1 = 2\gamma^2\omega_0/(1+K^2/2)$ with the undulator angular frequency $\omega_0 = 2\pi c/\lambda_U$ and the period of the undulator λ_U. Gathering the radiations from the individual periods, one finds that the intensity is proportional to

$$\sum_{m=1}^{N} \exp\left(2\pi i\left[m-1\right]\frac{\omega}{\omega_1}\right),$$

where N is the number of periods of the undulator. This implies that the intensity of the undulator radiation is strengthen at $\omega = n\,\omega_1$ with an integer n and that the spectrum has the harmonic structure.

A basic magnetic structure for the planer QPU can be realized by aligning positive and negative magnet poles alternately at the 1D quasi-lattice points. The mth quasi-periodic lattice point is represented as

$$\tilde{z}_m = m-(\tan\alpha-1)+(\tan\alpha-1)\left[\frac{\tan\alpha}{1+\tan\alpha}m+1\right],$$

where $\tan\alpha$ is the tangent of the inclination angle of a 1D quasi-lattice with rrespect to a 2D square lattice space. In this expression to represent the quasi-lattice point, $\tan\alpha$ should be some irrational number. As well as the PU, we can infer that the radiation intensity from a QPU contains the phase factor

$$\sum_{m=1}^{N'(=2N)} \exp\left(i\pi\frac{\omega}{\omega_1'}\left[\tilde{z}_m-\tilde{z}_1-\eta(m-1)\right]-i\pi m\right)$$

with N the number of the magnet poles. The fundamental radiation frequency of the QPU ω_1' is given by

$$\omega_1' = \frac{2\gamma^2\omega_0}{1+K^2}\tan\alpha$$

with $\omega_0 = (2\pi c)/(2d)$ the frequency of the QPU and d the length of the magnetic pole of the QPU. The factor $i\pi m$ reflects the fact that the magnet poles alternately change their polarities. Then it can be shown that the intensity spectrum possesses the bright peaks at

$$\frac{\omega}{\omega_1'} = k_{pq},$$

where k_{pq}'s are the irrational numbers designated by the Fibonacci integers p, q

$$k_{pq} = \left[2\left(p+q\frac{\tan\alpha}{1+\tan\alpha}\right)-1\right] / \left(\frac{1+\tan^2\alpha}{1+\tan\alpha}-\eta\right).$$

For the QPU with $\tan\alpha=1/\sqrt{5}$, $k_{10}=0.6234$, $k_{2\text{-}1}=1.4849$, $k_{3\text{-}2}=2.3464$, and so on. Thus the radiation spectrum from a QPU has the irrational harmonic structure and contains no rational higher harmonics.

To confirm the validity of the spectral formula, we compare the analytical computation with the numerical one. The radiation spectrum on axis from the QPU with $\tan\alpha=1/\sqrt{5}$ and $K=1$ is shown in Fig.1. The full circles in the figure represent the bright peaks given by the analytical calculation.

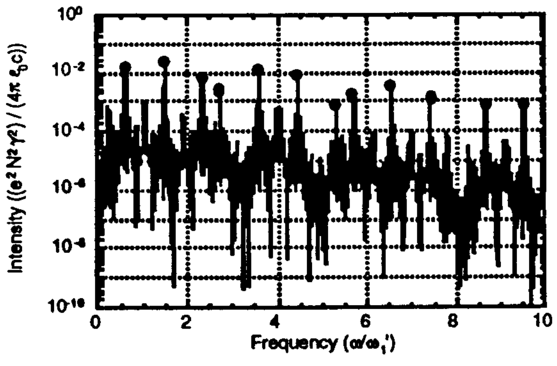

Fig.1. The radiation spectrum on axis from the QPU. The solid line represents the spectrum calculated numerically. The full circles correspond to the bright peaks given by the analytical computation.

Reference

[1] S. Hashimoto and S. Sasaki, JAERI-M Report 94-055 (1994). S. Sasaki, et al., in Proc. of Inter. Conf. of Synchrotron Radiation '94, New York, U.S.A., 1994.