Minimum Effective Emittance in Synchrotron Radiation Sources Composed of Modified Chasman Green Lattice

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From the viewpoint of the brilliance, a physical quantity to be optimized is not the natural emittance, but the effective emittance which represents the total spread of circulating beams at a source point in a median plane. To obtain a guideline on the design of light sources composed of Chasman Green (CG) lattice, we have derived the analytical representations for the minimum effective emittance and for the optimum distributions of betatron and dispersion functions within each bending magnet for the given parameters of the ring and insertion devices (IDs) [1].

The minimum effective emittance, $\varepsilon_{\text{eff min}}$, is given by

$$
\varepsilon_{\text{eff min}} = \frac{C_q \gamma^2 \theta_b^2 \sqrt{T(B)T(C)}}{2160 \rho J_x A_0 \sqrt{S_{opt} \theta_b + \theta_b^2}}.
$$

(1)

$T(z) = 16 \pi S_{opt}^2 + 24 \pi \theta_b S_{opt} + 9 \pi \theta_b^2 + 8 S_{opt} \rho^2 z.$

$$
A = \frac{2 \pi}{\rho} \sum_{i=1}^{N_{ID}} \frac{L_{ID,i}}{2 \rho_{ID,i}} B = \frac{4}{3 \pi} \sum_{i=1}^{N_{ID}} \frac{L_{ID,i}}{\rho_{ID,i}}.
$$

$$
C = 2 \frac{\pi J_x}{\rho^3 J_E} + \frac{(J_x + J_E) B}{J_E}.
$$

$$
S_{opt} = \sqrt{R_s} + \sqrt{R_d} - \frac{C_{12}}{3}, R_d = q = \sqrt{\gamma^2 + 4 p^2},
$$

$$
q = \frac{2 C_{12}}{27}, C_{12} = \frac{C_2}{C_1}, C_{14} = \frac{C_4}{C_1}, C_{14} = \frac{C_4}{C_1},
$$

$$
C_1 = 16 \left\{ 2 \pi \theta_b \left( \frac{B + C}{2} \right) + \frac{B C \rho^2}{2} + 4 \pi^2 \right\},
$$

$$
C_2 = 32 \pi \theta_b \left( 4 \pi + \rho^2 \left( \frac{B + C}{2} \right) \right),
$$

$$
C_3 = \pi \theta_b \left( 9 \rho^2 \left( \frac{B + C}{2} \right) + 84 \pi \right),
$$

where $C_q, \gamma, \theta_b, \rho, J_x$, and $J_E$ are respectively the quantum constant, Lorentz factor, the deflection angle of a bending magnet, the radius of curvature, the horizontal, and the longitudinal damping partition numbers. The symbols in Eq. (1), $N_{ID}, \rho_{ID,i}$, and $L_{ID,i}$ stand for respectively the total number of IDs installed, the radius of curvature of the peak field of the $i$th ID, and the length of the $i$th ID.

By using Eq. (1) and other equations in Ref. 1, $\varepsilon_{\text{eff min}}$ has been investigated numerically. We find: (a) In the case without IDs, $\varepsilon_{\text{eff min}}$ is about one half of the minimum natural emittance for the rigid CG lattice with the doubly achromatic condition. In the practical region where the minimum horizontal betatron function within each bending magnet is around 1 m, breaking the achromatic condition, i.e., the modified CG lattice is more advantageous for the low effective emittance compared with the rigid CG lattice. The effective emittance could be reduced by 55 - 60 %. (b) In the case with IDs, $\varepsilon_{\text{eff min}}$ and the optimum condition of the lattice functions in each bending magnet much depend on the parameters of IDs installed, that is, the number and the $K$ value. In the SPring-8 storage ring, breaking the achromatic condition is useful for reducing the effective emittance in the region where $K$ value is less than unity.

To obtain a general criterion to estimate which is better to obtain the low effective emittance, the achromatic or non-achromatic condition, we have presented a simple model to describe the phenomenon and derived the criterion. This criterion shows that breaking is useful when the excitation of a synchrotron oscillation due to the radiation from all insertion devices is smaller than a quarter of that due to the radiation from all bending magnets. Figure 1 shows two kinds of effective emittance for the SPring-8 storage ring as a function of an ID excitation parameter, the sum of $L_{ID,i}/\rho_{ID,i}$. As seen in Fig. 1, the criterion is useful to optimize the CG lattice for highly brilliant photon beams.

![Fig. 1 Effective emittance in the SPring-8 storage ring as a function of the ID excitation parameter. The open and closed symbols show respectively $\varepsilon_{\text{eff min}}$ and $\varepsilon_{\text{eff achro}}$. The circles, triangles, squares, and diamonds show respectively the $K$ values of 0.5, 1, 3, and 5.](image)

Reference