Estimation of Injected Beam Energy Based on Parameter Analysis of First Turn Horizontal Trajectory with Propagation Matrix

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In the case where only bending magnets are excited in a storage ring and electron beams are injected on the reference orbit of the ring by an on-axis injection, the trajectory for center of mass of injected electron beams, $\tilde{T}=(X, X', Y, Y')$ is expressed by injection errors and other parameters as,

$$\tilde{T} = (X, X', Y, Y') =$$

$$\tilde{f}\left(X_i, X'_i, Y_i, Y'_i, \Delta E_r, \frac{\Delta E}{E_0}, \Delta S_j, \Delta\theta_j, \frac{\Delta B_j}{B_0} \right), \quad (1)$$

where X, X', Y, and Y' are respectively the horizontal displacement, horizontal angle, vertical displacement, and vertical angle of the trajectory and the suffix i means those of the injection orbit. The symbols in Eq. (1), ΔE_r, $\frac{\Delta E}{E_0}$, ΔS_j, $\Delta\theta_j$, and $\frac{\Delta B_j}{B_0}$ are respectively the radiation loss, the energy deviation, the longitudinal misalignment, tilt error, and field error of the jth bending magnet. Since the X-Y coupling is small when quadrupole and sextupole magnets are off, Eq. (1) can be separated into two parts as,

$$\tilde{X}=(X, X')=\tilde{f}_1\left(X_i, X'_i, \Delta E_r, \frac{\Delta E}{E_0}, \Delta S_j, \frac{\Delta B_j}{B_0} \right), \quad (2)$$

$$\tilde{Y}=(Y, Y')=\tilde{f}_2(Y_i, Y'_i, \Delta\theta_j), \quad (3)$$

and Eq. (2) is used for the estimation of injected beam energy. If we can observe the first horizontal trajectory at numbers of position monitors, the energy deviation is obtained by solving the following equation with the least square method,

$$\begin{bmatrix} X_1 \\ \vdots \\ X_k \end{bmatrix} = \begin{bmatrix} \frac{\partial X_1}{\partial X_i} & \frac{\partial X_1}{\partial X'_i} & \frac{\partial X_1}{\partial(\Delta E/E_0)} \\ \vdots & \vdots & \vdots \\ \frac{\partial X_k}{\partial X_i} & \frac{\partial X_k}{\partial X'_i} & \frac{\partial X_k}{\partial(\Delta E/E_0)} \end{bmatrix} \cdot \begin{bmatrix} X_i \\ X'_i \\ \frac{\Delta E}{E_0} \end{bmatrix}, \quad (4)$$

Table 1. Simulation condition.

• Monitoring errors		
•• Alignment error:		rms. 0.1 mm
•• Dispersion of electronic circuit characteristics:		rms. 0.1 mm
•• Error due to noise:		rms. 1.0 mm
• Injection errors*)		
•• Position:	(hori.)	rms. 0.5 (+0.5) mm
	(vert)	rms. 0.5 (+0.3) mm
•• Angle:	(hori.)	rms. 0.1 (-0.1) mrad
	(vert)	rms. 0.1 (+0.2) mrad
• Longitudinal misalignment of bending magnets:		rms. 0.5 mm
• Remnant field errors:		
•• Quadrupole		0.2 %
•• Sextupole		0.2 %
• Monitor arrangement (shown in Fig. 1)		
•• Case-1:		12 monitors
•• Case-2:		10 monitors
•• Case-3:		8 monitors

*) The value within the parenthesis shows the systematic error.

where k stands for the number of monitors. The propagation matrix in Eq. (4) can be calculated by a particle tracking code.

To find the resolution of this method, we have simulated the first turn trajectory and estimated the energy deviation of injected beams with Eq. (4), taking account of the radiation loss at each bending magnet [1] and various practical errors. The simulation condition is listed in Table 1. Here, we have assumed that the 12 monitors shown in FIG.1 are usable in consideration of both the divergence of the injected beams due to natural emittance and the noise due to an electro-magnetic cascade induced by the electrons lost at the injection section.

Figure 2 shows the examples of horizontal and vertical trajectories simulated by the tracking code. The data are dispersed due to the fluctuation of injection errors and also jump discontinuously due to monitoring errors. As seen in FIG.2, the essence of this method is that the horizontal trajectory deviates systematically due to the energy deviation, $\Delta E/E_0$ at each bending magnet and this systematic deviation (pile-up effect) enables us to estimate $\Delta E/E_0$ precisely. The reason why the average of vertical trajectories doesn't vanish is that the simulation includes systematic injection errors.

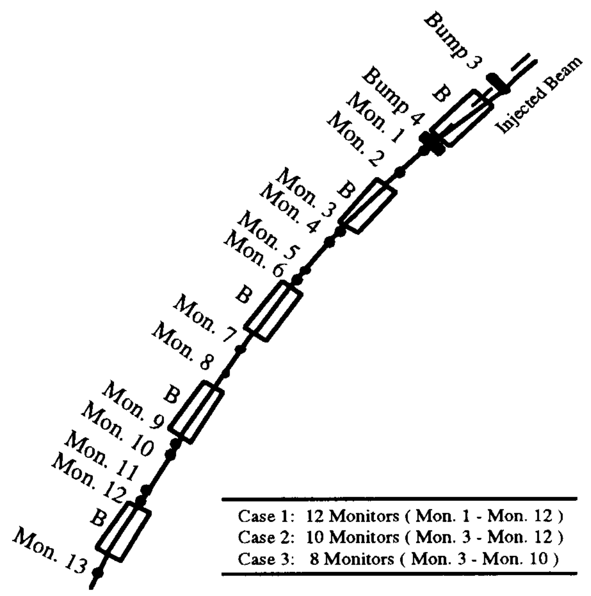

Fig. 1 Monitor arrangement used for the simulation. The symbols B, Mon., and Bump show respectively the bending magnet, the position monitor and the bump magnet.

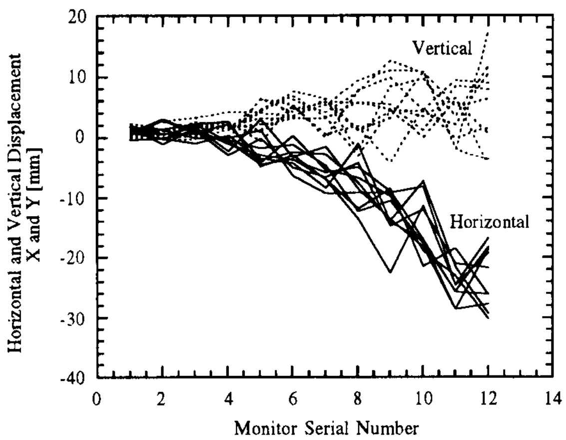

Fig. 2 Examples of horizontal and vertical trajectories or 10 different injection errors. The error condition is the same as in Table 1 and $\Delta E/E_0$ is -0.005. The abscissa represents the serial number which begins at the monitor just downstream from an injection point.

Figure 3 shows the resolution of $\Delta E/E_0$ as a function of the number of data N_a for averaging. Under the error condition listed in Table.1 and with the monitor arrangement of the Case-1, the resolution starts to saturate around $N_a = 10$ and the limit of the resolution is about 6×10^{-4}. The resolution also doesn't depend on $\Delta E/E_0$ down to $\Delta E/E_0$ of 5×10^{-4}. This result shows the non linearity of $\Delta E/E_0$ is negligible in the region where $|\Delta E/E_0| \le 0.005$.

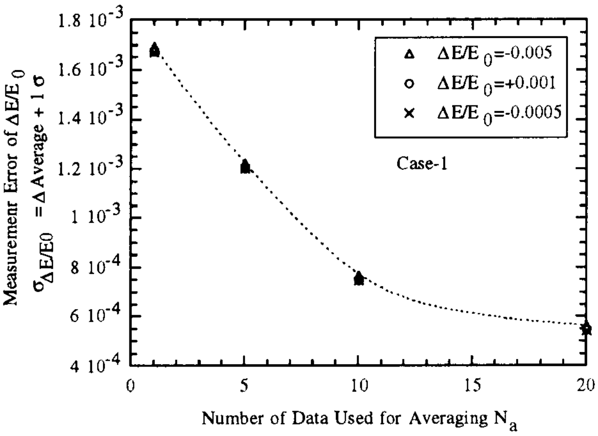

Fig. 3 Resolution of $\Delta E/E_0$ vs. number of data used for averaging by using $\Delta E/E_0$ as a parameter. The Case-1 is used as a monitor arrangement.

Figure 4 shows the effect of monitor arrangement on the resolution. We find that the Case-1 composed of 8 monitors gives a rather bad resolution compared with the Case-2 and Case-3, especially at small number of data for averaging.

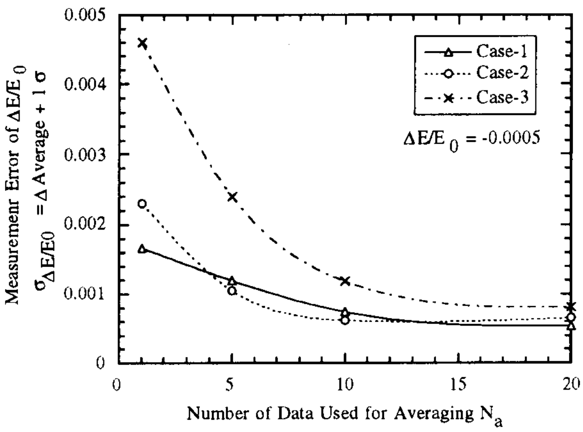

Fig. 4 Resolution of $\Delta E/E_0$ vs. number of data used for averaging. The energy deviation of -0.0005 is used for the simulation.

Reference

[1] H. Tanaka et al., This report, p.***.