Estimation of Local Betatron Functions and Phase Advances of Betatron Oscillations

Hitoshi TANAKA

SPring-8, Kamigori, Ako-gun, Hyogo 678-12, Japan

By measuring tune shifts of horizontal and vertical betatron oscillations caused by the tuning the strength of each quadrupole magnet, we can simply estimate the distribution of betatron functions. However, at the first stage of the ring operation, we can't tune the strength of each quadrupole independently. We have therefore studied another method based on the propagation of a single kick. The merit of this method is to obtain both the betatron function and phase advances consistently. Closed orbit distortion (COD) generated by the single kick of δ is observed at the ith monitor as

$$COD_{z,m(i)} =$$

$$\delta \cdot \frac{\sqrt{\beta_{z,m(i)}\beta_{z,s(j)}}}{2 \cdot \sin(\pi \nu_z)} \cdot \cos\left(\pi \nu_z - |\varphi_{z,m(i)} - \varphi_{z,s(j)}|\right), \quad z = x,\ y, \quad (1)$$

where ν_z, β_z, and φ are respectively the ideal betatron tune, the ideal betatron function, and the ideal phase advance of a betatron oscillation. The suffices x and y represent respectively the horizontal and vertical planes and m(i) and s(j) the ith monitor and jth steerer. By assuming that the modulations of horizontal and vertical betatron oscillations are small, the difference of the measured COD from a theoretical value is expressed by

$$\Gamma_{z,q(m(i),s(j))} =$$

$$\frac{COD_{z,m(i)}}{\delta} - \frac{\sqrt{\beta_{z,m(i)}\beta_{z,s(j)}}}{2 \cdot \sin(\pi \nu_z)} \cdot \cos\left(\pi \nu_z - |\varphi_{z,m(i)} - \varphi_{z,s(j)}|\right) =$$

$$\frac{\sqrt{\beta_{z,m(i)}\beta_{z,s(j)}}}{4 \cdot \sin(\pi \nu_z)} \cdot \cos\left(\pi \nu_z - |\varphi_{z,m(i)} - \varphi_{z,s(j)}|\right) \cdot$$

$$\cdot \left(\frac{\Delta\beta_{z,m(i)}}{\beta_{z,m(i)}} + \frac{\Delta\beta_{z,s(j)}}{\beta_{z,s(j)}}\right) + \mathrm{Sign}(\varphi_{z,m(i)} - \varphi_{z,s(j)}) \cdot \frac{\sqrt{\beta_{z,m(i)}\beta_{z,s(j)}}}{2 \cdot \sin(\pi \nu_z)} \cdot$$

$$\cdot \sin\left(\pi \nu_z - |\varphi_{z,m(i)} - \varphi_{z,s(j)}|\right) \cdot \left(\Delta\varphi_{z,m(i)} - \Delta\varphi_{z,s(j)}\right), \quad (2)$$

where $\Delta\beta_{z,m(i)}$ and $\Delta\beta_{z,s(j)}$ are respectively the modulations of the betatron function at ith monitor and at jth steerer, and $\Delta\varphi_{z,m(i)}$ and $\Delta\varphi_{z,s(j)}$ the modulations of the phase advance at ith monitor and at jth steerer. If we measure the propagation of a single kick with N_m monitors changing the steerer from 1 to N_s, we can obtain the following parallel linear equations. Although Eq. (3) is solved straight-forward by the least square method, we note the

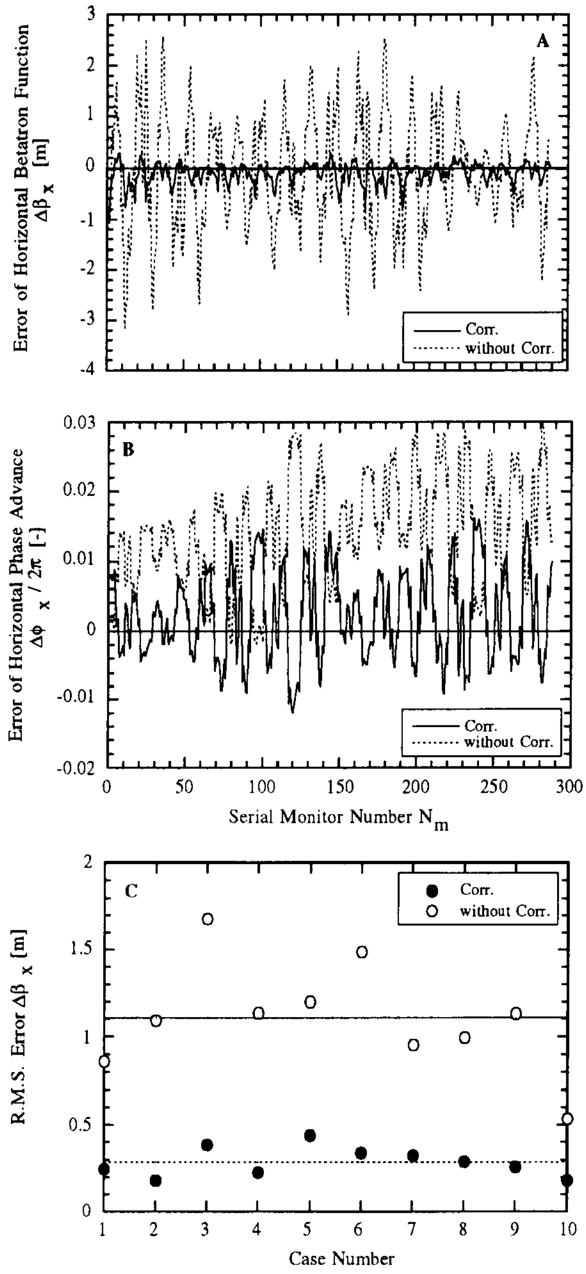

Fig. 1 Distribution of estimation error for the horizontal betatron function (A), that for the horizontal phase advance (B), and statistical distribution of estimation error for the horizontal betatron function (C). Rms. sextupole misalignment of 0.1 mm is used as an error source.

$$
\begin{bmatrix} \Gamma_{z,1} \\ \vdots \\ \Gamma_{z,\,Nm\times Ns} \end{bmatrix} =
$$

$$
\begin{bmatrix} \partial\Gamma_{z,1}/\partial\Delta_{z,1} & \dots & \partial\Gamma_{z,1}/\partial\Delta_{z,\mu} \\ \vdots & \vdots & \vdots \\ \partial\Gamma_{z,\,Nm\times Ns}/\partial\Delta_{z,1} & \dots & \partial\Gamma_{z,\,Nm\times Ns}/\partial\Delta_{z,\mu} \end{bmatrix} \cdot \begin{bmatrix} \Delta_{z,1} \\ \vdots \\ \Delta_{z,\mu} \end{bmatrix}, \quad (3)
$$

$\mu = 2 \times (Ns + Nm)$,
$\Delta_{z,\mu} = \Delta\beta_{z,m(i)}$ or $\Delta\beta_{z,s(j)}$ or $\Delta\varphi_{z,m(i)}$ or $\Delta\varphi_{z,s(j)}$.

followings: (a) Since the coefficient matrix is huge (400 MB for the SPring-8 storage ring), we need a sophisticated algorithm to reduce the memory volume keeping accuracy. (b) To reduce the calculation error, reasonable constraints should be introduced to the system to be solved.

We have been optimizing the algorithm by using the simulation code. As a tentative result, we have rms. error of 0.3 m and 1 deg. respectively for the betatron function and phase advance. Figure 1 shows some of simulation results obtained.