Estimation Scheme of Longitudinal and Transverse Impedance

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1 Introduction

The scheme to estimate longitudinal and transverse impedance of the SPring-8 storage ring is mentioned. The impedance is estimated with the simulation by MAFIA based on the several models of wake functions and impedance.

In this estimation, instead of the first moment of transverse impedance, the first moment of longitudinal wake potential is used to obtain transverse impedance using Panofsky-Wenzel theorem.

2 Longitudinal Impedance

With simulation by MAFIA, we can easily obtain the shape and parameters of wake potentials such as loss parameter k_l and the maximum and minimum of wake functions $W^{\parallel}_{min}$, $W^{\parallel}_{max}$ or wake potentials $V^{\parallel}_{min}$, $V^{\parallel}_{max}$.

These parameters of wake functions are compared with the wake functions based on several models of impedance which are shown in Table 1 and Fig. 1[1,2,3].

3 Transverse Impedance

By the Panofsky-Wenzel theorem[1], m-th moment of longitudinal and transverse impedance, $Z^{\parallel}_m(\omega)$ and $Z^{\perp}_m(\omega)$, respectively, have the relation ;

$$Z^{\perp}{}_m(\omega) = \frac{c}{\omega} Z^{\parallel}{}_m(\omega) \quad . \qquad (1)$$

On the other hand, with dimensional analysis[1], we have a relation between $Z^{\parallel}_0(\omega)$ and $Z^{\perp}_1(\omega)$;

$$Z^{\perp}{}_1(\omega) \approx \frac{2c}{d^2\omega} Z^{\parallel}{}_0(\omega) \qquad (2)$$

,where d is a constant of the dimension of length and $\omega=n\omega_{rev}$; ω_{rev} is the angular frequency of revolution.

For the resistive-wall impedance and cavity impedance based on the diffraction model, this relation is strictly valid if we set d=b.

With equation (1) and (2), we can get the relation

$$Z^{\parallel}{}_1(z) \approx \frac{2}{d^2} Z^{\parallel}{}_0(z) \quad . \qquad (3)$$

The corresponding relation between wake functions $W_1'(z)$ and $W_0'(z)$ is

$$W_1'(z) \approx \frac{2}{d^2} W_0'(z) \qquad (4)$$

In terms of the wake potentials at (x,y)=(x,0), produced by a charge passing at (x',y')=(a,0), this relation is

$$V^{\parallel}{}_1(z) \approx \frac{2ax}{d^2} V^{\parallel}{}_0(z) \quad . \qquad (5)$$

If we set a=x=b, this becomes

$$V^{\parallel}{}_1(z) \approx 2\frac{b^2}{d^2} V^{\parallel}{}_0(z) \quad . \qquad (6)$$

And if we set d=b, where b is the radius of a beam chamber, the equation (6) becomes

$$V^{\parallel}{}_1(z) \approx 2V^{\parallel}{}_0(z). \qquad (7)$$

Equation (6) or (7) show that, with comparing the shape of $V^{\parallel}_0(z)$ and $V^{\parallel}_1(z)$, we can test the validity of equation (2) and find the value of d. This comparison is very intuitive because the function shapes and magnitude of $V^{\parallel}_0=-W_0'$ and $V^{\parallel}_1=-b^2W_1'$ are almost same.

For three-dimensional structures, $Z^{\perp}_1$ are defined for each transverse direction x and y. We assume that the corresponding equation to Eq. (2)

$$Z^{\perp}_{x,y1}(\omega) \approx \frac{2c}{d_{x,y}{}^2\omega} Z^{\parallel}{}_0(\omega)$$

is good approximation even in three-dimensional structure, where d_x and d_y is defined for each direction to x and y, individually.

The same impedance models can applied to obtain $Z^{\parallel}_1$ from wake potentials $V^{\parallel}_1$. Then $Z^{\perp}_1$ can be obtained from $Z^{\parallel}_1$ with eq.(1).

Table 1. Models of longitudinal impedance $Z^{\parallel}_0(\omega)$ and $Z^{\parallel}_0(\omega)$ expressed with $W^{\parallel}_{max}$, or k_l [2,3]

Model	Longitudinal impedance $Z^{\parallel}_0$		
	cavitylike	inductive	resistive
Frequency dependence	$Z_c \frac{1+i}{\sqrt{\omega}}$	$-i\omega L$	R
With $W^{\parallel}_{max}$	$\frac{1}{1.2824}\frac{\pi}{2}\sqrt{\frac{\sigma}{c}}W^{\parallel}{}_{max}\frac{1+i}{\sqrt{\omega}}$	$-i\omega\sqrt{2\pi e}\left(\frac{\sigma}{c}\right)^2 W^{\parallel}{}_{max}$	$\sqrt{2\pi}\frac{\sigma}{c}W^{\parallel}{}_{max}$
With k_l	$\frac{1}{\frac{\Gamma(1/4)}{4}\frac{2}{\pi}}\sqrt{\frac{\sigma}{c}}k_l\frac{1+i}{\sqrt{\omega}}$	-	$2\sqrt{\pi}\frac{\sigma}{c}k_l$

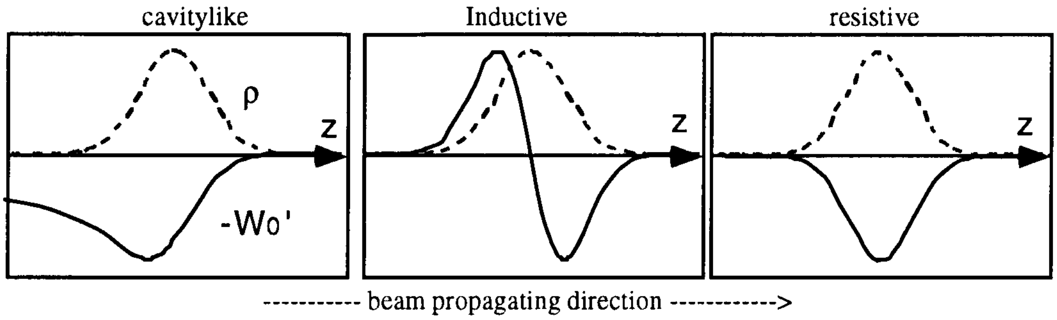

Fig. 1. Longitudinal wake functions for cavitylike, inductive and resistive elements.
solid line : Longitudinal wake potential $V_0 = -W_0'$
dashed line : Charge density of bunch (Gaussian) ρ

4 Theoretical Equations

We also uses theoretical equations shown in Table 2.

Table 2. Theoretical longitudinal impedance [2,3]

Model	Theoretical impedance $Z^{\parallel}_0(\omega)$
cavitylike	$\frac{Z_0}{2\pi}\frac{1}{a}\sqrt{\frac{cg}{\pi}}\frac{1+i}{\sqrt{\omega}}$
shallow groove	$-i\omega\frac{Z_0}{2\pi c}\frac{g(b'-b)}{b}$
shallow transitions	$-i\omega\frac{3Z_0}{2\pi c}\frac{b(b'-b)^2}{b^2}\left(\frac{2\theta}{\pi}\right)^{1/2}$
resistive wall	$Z_0\frac{1-i}{2}\frac{\delta}{b}, \quad \delta=\sqrt{2/\omega\mu\sigma}$
synchrotron radiation	$\lvert Z^{\parallel}\rvert = 300\frac{b}{c}\lvert\omega\rvert$

5 Simulation for Convex Shape

The ID sections and the RF absorbers have convex shape like the shape I in Fig. 2 and it is difficult to get stable simulatin result for these convex shapes because the indirect method can not be applied in such shapes with MAFIA T2 and T3.

But the simulations show that the impedance of the shape II in Fig. 2, to which the indirect method can be applied, have weak dependence on the length L. This result means that the interference between the wakes of the section A and the section B are small. Based on this fact, it is assumed that the interference is also small in the shape I. Hence the shape II is used for simulation instead of the shape I in the simulations for ID sections and RF absorbers.

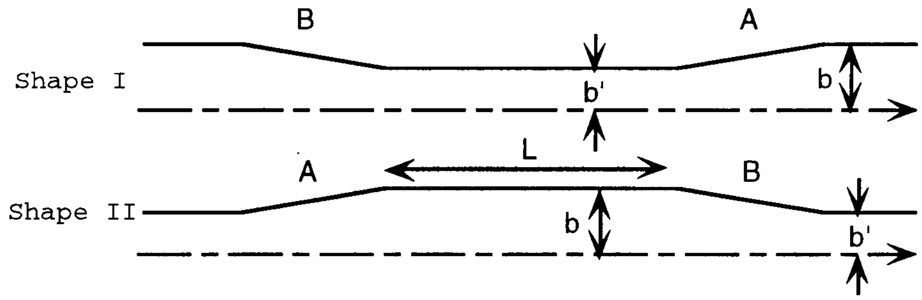

Fig. 2. The shape for simulation.
Shape I : actual convex shape at ID sections and absorbers.
Shape II : the concave shape for the simulation.

6 Conclusion

The method to estimate longitudinal and transverse impedance is shown. For transverse impedance estimation, we first estimate $Z^{\parallel}_1$ then we get $Z^{\perp}_1$ from $Z^{\parallel}_1$ using Panofsky-Wenzel theorem and this method is very intuitive because of the wake potential $V^{\parallel}_0$ and $V^{\parallel}_1$ produced by $Z^{\parallel}_1$ and $Z^{\parallel}_0$ have similar shapes and magnitude.

The longitudinal and transverse impedance of the SPring-8 storage ring is estimated with this scheme[4].

References

[1] A. W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators," 1993, John-Wiley & Sons Inc.

[2] K. L. F. Bane, SLAC-PUB-5177(1990)

[3] S. Heifets, SLAC/AP-93(1992).

[4] T.Nakamura, "Longitudinal and Transverse Impedance of the SPring-8 Storage Ring," in this report.