Characteristics of Impedance of Small Grooves

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1 Introduction

Characteristics of the impedance of groove is studied with the simulation. Vacuum chambers of rings have many small discontinuities at flanges, weldments and so on. For impedance issues, it is very important to understand the characteristics of impedance of such element to get the region where the estimation is valid.

The impedance itself does not depend on the bunch length but the bunch samples the impedance by its frequency spectrum and the impedance seen by the bunch is depends on the bunch length. Three model impedance are assumed for the impedance seen by the bunch.

2 Test Groove

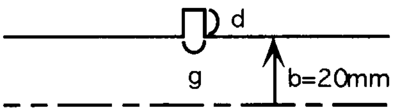

Fig.1. The shape of a test groove.

To analyze the impedance, a test groove shown in Fig.1 is used to get wake functions by MAFIA T2, the solver for time-dependent system in cylindrical symmetric structure.

3 Wake Functions

The wake potential for a groove, g=1mm and d=10mm, is shown in Fig. 2. This wake potential shows that the impedance is cavitylike or resistive for a short bunch;σ_z << d and is inductive for a long bunch;σ_z >> d.

Qualitative explanation is as follows; the electromagnetic field(EM-field) produced by the bunch passage at the gap has resistive or cavitylike wake. Then this field propagate to the bottom of a groove. The electric field changes the direction at the bottom and reflects back to the beam pipe. At the beam pipe, this EM-field produce the wake opposite to the first wake. Then it reflects again to the bottom. For a longer bunch than the depth d, the impedance is inductive because such a bunch feels this reflected EM-field which has the opposite sign to the first one. This is the same mechanism as in a differential circuit with delay line. In the following, this is confirmed quantitatively.

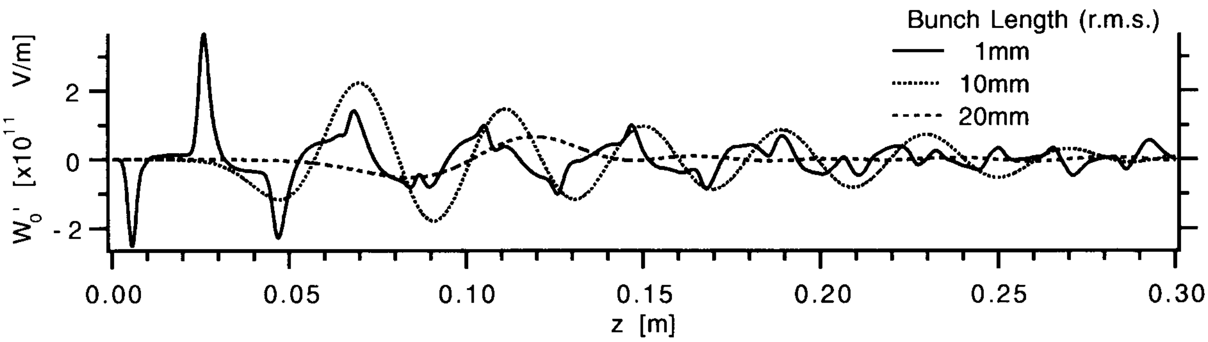

Fig. 2. A wake function of a groove of 1mm(gap)×10mm(depth) for different bunch length. Bunch centers are at z=0.005m, 0.05m and 0.1m for σ_z=1mm, 10mm and 20mm, respectively.

4 Model Impedance

The three models of impedance are assumed and are shown in Table 1[1,2].

Table 1. Impedance models

Model	$Z(\omega)$	Maximum of $W^{\parallel}$ ($W^{\parallel}_{max}$)	Loss Parameter (k_l)
cavitylike	$Z_c\frac{1+i}{\sqrt{\omega}}$	$Z_c \times 1.28\frac{2}{\pi}\sqrt{\frac{c}{\sigma}}$	$Zc\frac{\Gamma(1/4)}{4}\frac{2}{\pi}\sqrt{\frac{c}{\sigma}}$
resistive	R	$R\frac{1}{\sqrt{2\pi}}\frac{c}{\sigma}$	$R\frac{c}{2\sqrt{\pi}\sigma}$
inductive	$-i\omega L$	$L\frac{1}{\sqrt{2\pi e}}\left(\frac{c}{\sigma}\right)^2$	0

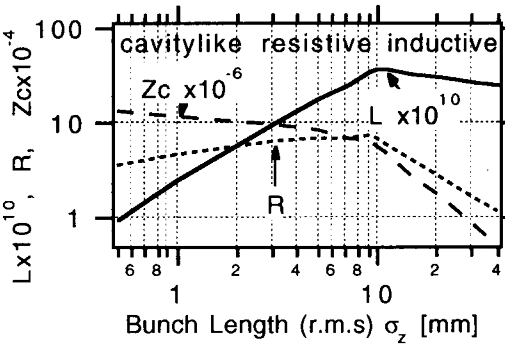

Fig. 3. L, R, Z_c of the groove, g×d=3mm×10mm, obtained from $W^{\parallel}_{max}$ The flat region is where the impedance model is valid.

5 Simulation Result

With the simulation, the dependence of L, R and Z_c on the bunch length is obtained from $W^{\parallel}_{max}$ using the equations in Table 1 and is shown in Fig.3.

This shows that the impedance seen by the bunch is cavitylike for $\sigma_z<d$, inductive for $\sigma_z>d$ and resistive for $g<\sigma_z<d$. Table 2 shows that the value of impedance based on each model is coincide at the frequency $\omega=c/d$ where is the boundary of region of each model. This also confirm the validity of the impedance models assumed here.

Table 2. Impedance obtaind for a groove $g\times d$=3mm×10mm with simulation

Model	Z	L, R, Zc	$\lvert Z(\omega=c/d)\rvert$
inductive	$-i\omega L$	$L=-2.45\times10^{-10}$ (at σ_z = 40mm)	7.51
resistive	R	R=7.03 (at σ_z = 9mm)	7.03
cavitylike	$Z_c\frac{1+i}{\sqrt{\omega}}$	$Z_c=1.25\times10^6$ (at σ_z = 0.5mm)	7.24

The dependence of $W^{\parallel}_{max}$ on bunch length based on the model impedance is shown in Table 1 and the simulation results are shown in Fig. 4, where $W^{\parallel}_{max}\sigma_z$ is shown instead of $W^{\parallel}_{max}$. They shows that the model impedance is good approximation for each region. And in Fig. 4, the dependence of $W^{\parallel}_{max}$ on gap size g is, $W^{\parallel}_{max}\propto\sqrt{g}$, in cavitylike region, which is consistent of the result with the diffraction model, and $W^{\parallel}_{max}\propto g$ in resistive and inductive region, which is consistent to usual low frequency model of small groove[1]. And W_0' does not depend on the depth of the groove in cavitylike and resistive region because the bunch passes the groove before the reflected field come back. At the inductive region, $W^{\parallel}_{max}\propto d$ which is consistent with the low frequency model of small groove[1].

Fig. 5 shows that the loss parameter obtained by the simulation. This result also shows the validity of the impedance models.

In both cases, the point which divide inductive region and cavitylike region is σ_z~d which is the depth of the groove and is and the point which divide resistive and cavitylike region is at $\sigma_z<<d$ and σ_z~g.

6 Impedance of a Groove

The impedance of a groove obtained by above analysis is shown in Figure 6. It is cavitylike for $\sigma_z<g$ and $\sigma_z<d$, is inductive for $d<\sigma_z$ and is resistive for $g<\sigma_z<d$.

7 Conclusion

The impedance of a groove is analyzed with the simulation. For long bunch $\sigma_z>d$, the impedance is inductive and for short bunch whose $\sigma_z<d$, the impedance is resistive or cavitylike and the bunch feels much higher loss parameter. And for a ring with positive compaction factor, these resistive and cavitylike wake shorten the bunch length and bunch become shorter and shorter until microwave instabilities occur. On the other hand for long bunch, the inductive wake lengthen the bunch and the bunch become longer and longer. For a ring with negative compaction factor, the situation is opposite and there is a balanced point in bunch length.

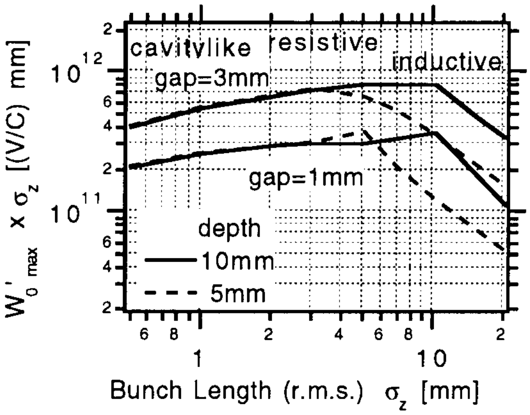

Fig. 4. The dependence of $W_{0max}'\ \sigma_z$ on bunch length σ_z. $W_{0\,max}'\sigma_z\propto\sqrt{\sigma_z}$ in cavity like region, $W_{0max}'\sigma_z\propto$const. in resistive region and $W_{0\,max}'\sigma_z\propto1/\sigma_z$ in inductive region.

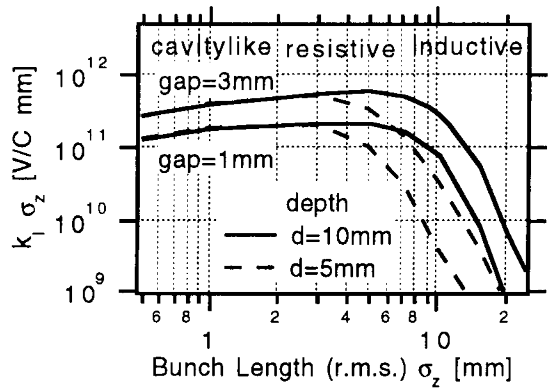

Fig. 5. loss parameter $k_l\times$bunch length σ_z vs. σ_z Regarding on Table 1, $k_l\sigma_z\propto\sqrt{\sigma_z}$ in cavitylike region, $k_l\sigma_z\propto$const. in resistive regiôn and $k_l\sigma_z\propto0$ in inductive region.

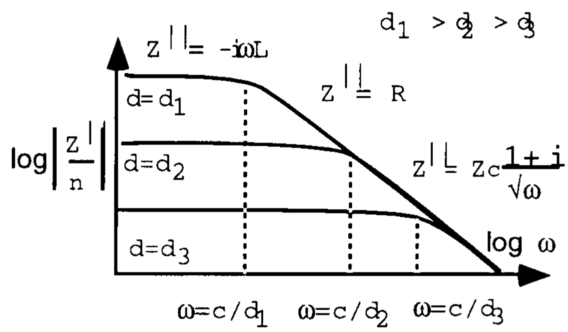

Fig. 6. The impedance of grooves of different depth d.

References

[1] K. L. F. Bane, SLAC-PUB-5177(1990)
[2] S. Heifets, SLAC/AP-93(1992).