The Coupled-Bunch Instabilities Simulation Code CISR - Longitudinal Motion -

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1 Introduction

The simulation code CISR(Coupled-Bunch Instabilities in Storage Ring) which simulates coupled-bunch instabilities caused by the resonator type impedance was developed. In this report, the treatment of longitudinal motion is described. For that of transverse motion, see ref.[1].

2 Evolution of Wake Voltage

In this code, each electron bunch is composed of superparticles. As shown in Fig.1, the bunch time spacing is $T_b = T_0 / M$ where T_0 is the revolution frequency and M is the number of bunches. The suffix i, j shows that it is the parameter of the j-th particle in i-th bunch. For example, the charge of the j-th particle in i-th bunch is $q_{i,j}$. Its arrival time at an impedance in n-th revolution is $\tau^n_{i,j}$, and its energy shift $\delta=\Delta E/E_0$ is $\delta^n_{i,j}$.

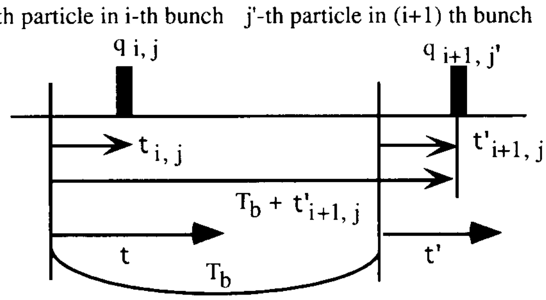

Fig. 1. Definition of parameters

The longitudinal impedance of a resonator have the form,

$$Z^{\parallel} = \frac{R^{\parallel}}{1 + i Q^{\parallel}\left(\frac{\omega^{\parallel}}{\omega} - \frac{\omega}{\omega^{\parallel}}\right)}$$

and its wake voltage $V(t)$ produced by charge q is[2]

$$V^{\parallel}(t) = -q\,\frac{R^{\parallel}}{Q^{\parallel}}\,\omega^{\parallel}\,e^{-\alpha^{\parallel} t}\left(\cos \overline{\omega}^{\parallel} t - \frac{\alpha^{\parallel}}{\overline{\omega}^{\parallel}} \sin \overline{\omega}^{\parallel} t\right)$$
$$= \mathrm{Re}\left[\tilde{V}^{\parallel} e^{i\lambda^{\parallel} t}\right]$$

where $\tilde{V}^{\parallel} = -q\,\frac{R^{\parallel}}{Q^{\parallel}}\,\omega^{\parallel}\left(1 + i\frac{\alpha^{\parallel}}{\overline{\omega}^{\parallel}}\right)$, $\lambda^{\parallel} = i\,\alpha^{\parallel} + \overline{\omega}^{\parallel}$,

$\alpha^{\parallel} = \frac{\omega^{\parallel}}{2Q^{\parallel}}$ and $\overline{\omega}^{\parallel} = \sqrt{\omega^{\parallel 2} - \alpha^{\parallel 2}}$.

The time t and t' are defined as Fig. 1. If the wake voltage between the i-th and (i+1)-th bunch is assumed to be

$$V^{\parallel}_i(t) = \mathrm{Re}\left[\tilde{V}^{\parallel}_i e^{i\lambda^{\parallel} t}\right],$$

the wake voltage between the (i+1)-th and (i+2)th bunch can be expressed as

$$V^{\parallel}_{i+1}(t') = \mathrm{Re}\left[\tilde{V}^{\parallel}_{i+1} e^{i\lambda^{\parallel} t'}\right]$$
$$= \mathrm{Re}\left[\tilde{V}^{\parallel}_i e^{i\lambda^{\parallel} t}\right] + \mathrm{Re}\left[\sum_{j=1}^{N_{p\,i+1}} \tilde{V}^{\parallel\,'}_{i+1,j}\, e^{i\lambda^{\parallel}(t' - t_{i+1,j})}\right]$$
$$= \mathrm{Re}\left[\tilde{V}^{\parallel}_i e^{i\lambda^{\parallel}(t' + T_b)} + \sum_{j=1}^{N_{p\,i+1}} \tilde{V}^{\parallel\,'}_{i+1,j}\, e^{i\lambda^{\parallel}(t' - t_{i+1,j})}\right]$$

where $N_{p,i}$ is the number of superparticle in i-th bunch. Using phasers, we have

$$\tilde{V}^{\parallel}_{i+1} = \tilde{V}^{\parallel}_i e^{i\lambda^{\parallel} T_b} + \sum_{j=1}^{N_{p\,i+1}} \tilde{V}^{\parallel\,'}_{i+1,j}\, e^{-i\lambda^{\parallel} t^n_{i+1,j}}$$

$$\tilde{V}^{\parallel\,'}_{i,j} = -q_{i,j}\,\frac{R^{\parallel}}{Q^{\parallel}}\,\omega^{\parallel}\left(1 + i\frac{\alpha^{\parallel}}{\overline{\omega}^{\parallel}}\right)$$

after the passage of (i+1) th bunch.

The change of the wake voltage amplitude inside of the bunch by each electron is neglected.

The energy gain of the electron by this wake is $e\,\mathrm{Re}\left[\tilde{V}^{\parallel}_i e^{i\lambda^{\parallel} T_b}\right]$.

The parasitic loss term by this impedance, $-\frac{1}{2}eV'_{i+1,j}$, is neglected because this term is negligible compared with total parasitic loss.

3 Acceleration Cavity

The beam loading $\tilde{V}_b$ changes in different filling pattern and this modulates the acceleration voltage V_c This modulation can produce synchrotron tune spread and this spread causes the Landau damping which can be used to suppress longitudinal coupled-bunch instabilities.

To simulate this effect, the acceleration cavities are treated as a system of a resonator impedance and the acceleration of voltage V_g driven by external power.

In the following, We assume that the nominal mode is a full-bunch mode and we change filling pattern in the condition optimized to this full-bunch mode.

For full-bunch mode, the external voltage V_g required is obtained as follows. The equivalent circuit of acceleration cavities is shown in Fig. 2 and its impedance is expressed as

$$Z_L = \frac{R_L}{1 + iQ_L\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} = R_L \cos\psi\, e^{-i\psi}$$

where $\tan\psi = Q_L\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)$, $R_L = \frac{R_C}{(1+\beta)}$, $Q_L = \frac{Q_C}{(1+\beta)}$.

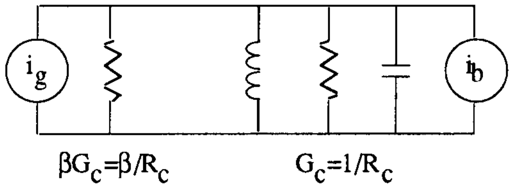

Figure 2. The equivalent circuit of a acceleration cavity.

This circuit is excited by an external current source i_g and a beam current i_b of frequency ω and they produce the voltage V_g *and* V_b, respectively.

The stored current is assumed to be

$$I(t) = \frac{I_0 T_0}{\sqrt{2\pi}\sigma_t}\sum_{k=-\infty}^{\infty} e^{-\frac{(t-kT_0)^2}{2\sigma_t^2}}$$

$$= I_0 + 2I_0 \sum_{n=-\infty}^{\infty} e^{-\frac{1}{2}(n\omega)^2\sigma_t^2} e^{in\omega}$$

and the Fourier component of the frequency of the beam ω is $\tilde{i}_b = 2I_0 e^{-\frac{1}{2}\omega^2\sigma_t^2} \simeq 2I_0$ where $\omega\sigma_t$ <<1 is assumed and the phase of current $\tilde{i}_b$ is set to be 0.

The beam loading voltage $\tilde{V}_b$ created by this current is

$$\tilde{V}_b = \tilde{i}_b Z_L = -i_b R_L \cos\psi\, e^{i\psi} .$$

and the cavity voltage $\tilde{V}_c = V_c e^{i\phi}$ is the sum of V_g and V_b as shown in Fig. 3,

$$\tilde{V}_c = \tilde{V}_g + \tilde{V}_b \text{ or } \tilde{V}_g = \tilde{V}_c - \tilde{V}_b$$

The designed cavity voltage V_c should be

$$eV_c\cos\phi = U,\quad \frac{eV_c}{U} = q$$

where U and q are the energy loss of stored electrons and the required overvoltage factor, respectively.

Then the required external voltage is

$$\tilde{V}_g = \tilde{V}_c - \tilde{V}_b = V_c e^{i\phi} + i_b R_L \cos\psi\, e^{i\psi} .$$

This voltage $\tilde{V}_g$ is applied to simulate the acceleration at cavities.

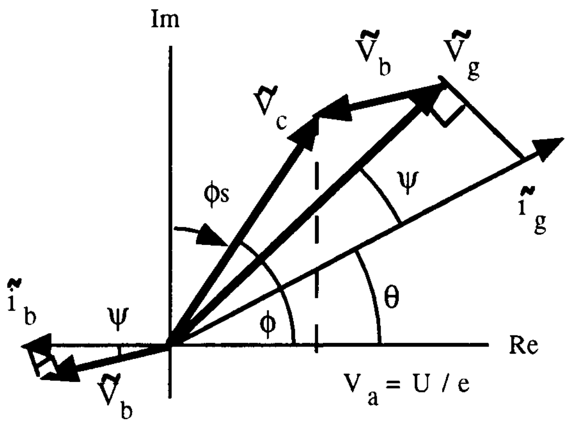

Fig. 3. Phaser diagram of an acceleration cavity

We want to add the discussion for the required power to drive an acceleration cavity at the voltage $\tilde{V}_g$ [3]. From the definition of i_g,

$$\tilde{V}_g = \tilde{i}_g Z = i_g e^{i\theta} R_L \cos\psi\, e^{i\psi}$$

, we get

$$i_g = \frac{V_g}{R_L\cos\psi} .$$

And from ref. [3], the power required to drive i_g is,

$$P_g = \frac{1}{8}\frac{i_g^2 R_c}{\beta} .$$

Then the P_g is

$$P_g = \frac{(1+\beta)^2}{8\beta}\frac{V_g^2}{R_c\cos^2\psi} .$$

If $\theta=\phi$, $\tan\psi = -\frac{i_b R_L}{V_c}\sin\phi$ and $\beta_o = 1+P_b/P_c$, we have minimum P_g,

$$P_g = \frac{V_c^2\beta_o}{2R_c}$$

where $P_c = V_c^2/2R_0$ is the wall loss in the cavity and $P_b = I_0 V_a$ is the power taken by the beam.

4 Simulation

The difference equations for particles and wake fields are as follows with the result above,

$$t_{i,j}^{n+1} = t_{i,j}^n + \alpha\,\delta_{i,j}^n T_0$$

$$\delta_{i,j}^{n+1} = \delta_{i,j}^n - \frac{U_0}{T_0 E_0}\left(1+\delta_{i,j}^n\right)^2$$

$$+\frac{eV_g}{E_0}\cos\left[\omega_g\left(T_{n,i} + t_{i,j}^{n+1}\right) + \phi_s\right]$$

$$+\frac{e}{E_0}\mathrm{Re}\left[\tilde{V}^{\parallel\,n+1}_{i-1} e^{i\lambda\left(T_b + t_{i,j}^{n+1}\right)}\right]$$

$$+\frac{e}{E_0}V_{ex}\left(T_{n,i} + t_{i,j}^{n+1}\right)$$

$$T_{n,i} = (nM + i)T_b$$

$$\tilde{V}^{\parallel\,n+1}_{i} = \tilde{V}^{\parallel\,n+1}_{i-1} e^{i\lambda^{\parallel}T_b} + \sum_{j=1}^{N_{p\,i}} \tilde{V}^{\parallel\prime}_{i,j} e^{-i\lambda^{\parallel}t_{i,j}^{n+1}}$$

$$\tilde{V}^{\parallel\prime}_{i,j} = -q_{i,j}\frac{R^{\parallel}}{Q^{\parallel}}\omega^{\parallel}\left(1 + i\frac{\alpha^{\parallel}}{\omega^{\parallel}}\right) .$$

Here, V_{ex} is the voltage of devices such as f+f_0 cavities.

5 Conclusion

This simulation code of multi-bunch instabilities for resonator-type impedance, CISR, was developed and this code also treat beam loading effect in acceleration cavities. CISR was developed in C++ in object-oriented programming which makes it easier to develop and modify the code.

References

[1] T. Nakamura, "The Coupled-Bunch Instabilities Simulation Code CISR - Transverse Motion -," In this report.

[2] A. W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators," 1993, John-Wiley & Sons Inc.

[3] P. B. Wilson, SLAC-PUB-2884,1991.