Study on Stability Limit of a Single Particle Motion due to RF Sextupole Magnets

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1. Introduction

RF Sextupole magnets which are modulated with the frequency of a synchrotron oscillation, ~ 2 kHz for the SPring-8 storage ring, was proposed by Nakamura [1] to cure coupled and uncoupled (single) bunch instabilities. Since the strength modulation of several RF sextupole magnets potentially makes single particle motions unstable, we have studied on the stability limit of a single particle motion due to the RF sextupoles.

2. Required strength modulation

For strong damping on coherent beam motions, large tune spread is necessary, which means the large strength modulation (chromaticity modulation), because the tune spread due to the RF sextupoles is proportional to the amplitude of the modulation. [1] To obtain the vertical tune spread of a few $\times 10^{-3}$ in the storage ring, we need from two to six as residual vertical chromaticity, $\Delta\xi_y$. Figure 1 shows the sextupole strength as a function of the number of sextupoles N_p periodically installed. In the case where $\Delta\xi_y = 5$ and $N_p = 4$, the amplitude of the modulation is estimated at ~ 3.5 m^{-2}, which reaches a half of the strength of focusing sextupoles for the linear chromaticity correction.

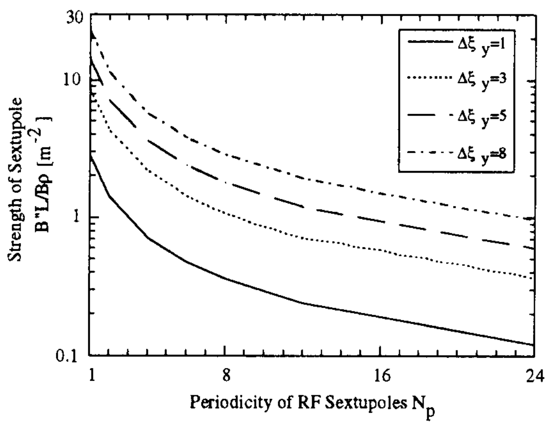

Fig. 1. Sextupole strength vs. number of sextupoles periodically installed. Vertical chromaticity is used as a parameter. We assume 0.4 and 11 m respectively as horizontal dispersion and betatron functions at the RF sextupoles.

3. Limit due to tune-shifts

Non zero chromaticity modulates the betatron tunes of each particle coupling with momentum spread. Since these tune shifts excite half integer resonances near the operation point, the magnitude of chromaticity modulation is limited.

In the storage ring, the horizontal and the vertical tunes are a few thousands times larger than the synchrotron tune. We can thus treat the system as decoupled and use the method in Ref. 2. The stability criterion for the half integer resonance, $2\nu_x = N$ is given by

$$\left|\delta + \frac{J_{z,0}}{4\pi}\right| > \frac{|J_{z,N}|}{4\pi}, \qquad (1)$$

$$J_{z,N} = \int_0^C ds \left(\frac{dp}{p_0}\right) \cdot \eta_x \cdot \frac{\Delta B_y''}{B\rho} \cdot \beta_z \cdot \mathrm{Exp}[-iN\phi_z(s)]\ ,\ z = x, y\ ,$$

where η_x, dp/p_0, δ, β_z, $B\rho$, $\Delta B_y^{\cdot}$, and ϕ_z stand for respectively the horizontal dispersion function, the momentum shift from a nominal momentum, the distance from a half integer resonance, the betatron function, magnet rigidity, the modulated strength of the RF sextupoles, the betatron phase advance. The suffices, x and y denote respectively the horizontal and the vertical planes. Figure 2 shows the stability map of the storage ring for the half integer resonances, assuming that the momentum shift has the same value as the equilibrium spread. The stable region enlarges as increasing the periodicity. At the small periodicity, the residual vertical chromaticity must be less than eight.

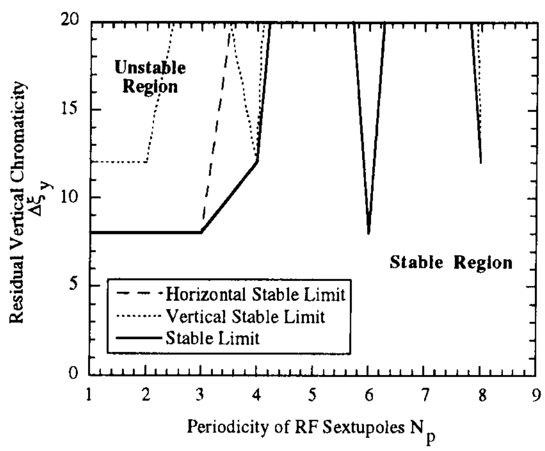

Fig. 2. Stability map of SPring-8 storage ring for the half integer resonances. Sextupole strength is determined assuming the same condition as in FIG.1.

To estimate the lifetime, we can use the equation for the quantum lifetime τ_q with an aperture limit, [3] which is expressed by

$$\tau_q = \tau_s \cdot \mathrm{Exp}\left(\frac{W^2}{2\delta^2_{p/p_0}} \right), \qquad (2)$$

where τ_s, $\delta_{p/p0}$, and W stand for the longitudinal damping time, the equilibrium momentum spread, and the momentum shift required to hit the nearest half integer resonance. Since the damping time of the storage ring is 4.15 msec, we see that W must be larger than $3.3\times\delta_{p/p0}$ to keep the lifetime longer than 10 hours. Therefore, by rescaling the result shown in FIG.2 ($\delta_{p/p0}$ is taken as $dp/p0$) by factor three, the possible chromaticity modulation is found to be within ± 2.5 under keeping the lifetime of 10 hours.

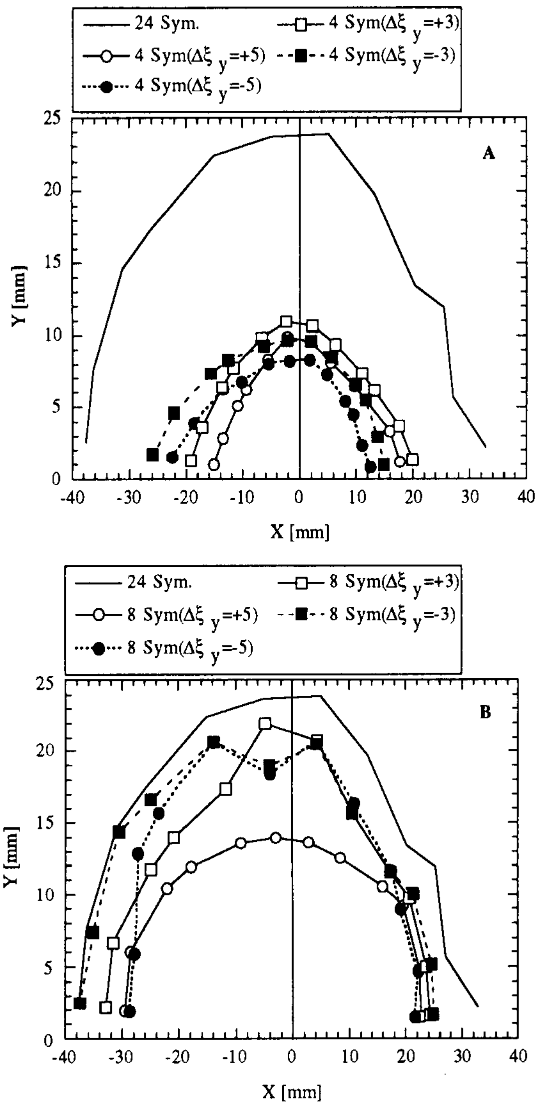

Fig. 3. Dynamic apertures for the rings where the RF sextupoles are installed with four fold symmetry (A) and eight fold symmetry (B). The calculation is performed without magnetic errors and the observation point is the center of a high beta straight section. The number of revolutions is 500.

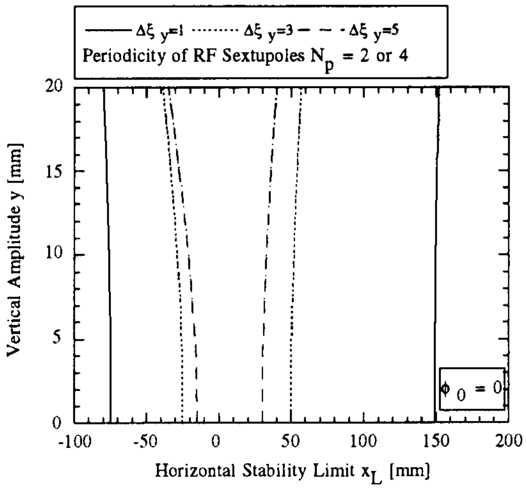

Fig. 4. Stability limit due to the resonance of $\nu_x = 52$ for the storage rings with two fold and four fold symmetries by using vertical chromaticity as a parameter. The symbol, ϕ_0 denotes the relative phase rotation of an unstable fixed point.

4. Limit due to nonlinear resonances (dynamic aperture)

We can check the effect of multiple nonlinear resonances on the transverse stability by particle tracking. Figure 3 shows the dynamic apertures for the storage rings where the RF sextupoles are installed with four fold and eight fold symmetries.

The arrangement with a four fold symmetry seems not to be sufficiently stable to operate the storage ring with large chromaticity. In the case of an eight fold symmetry, we don't see the remarkable reduction of the dynamic aperture.

The mechanism of this stability limit can be investigated by the resonance analysis in Ref. 4 which gives the limit caused by the first order of sextupole strength, that is, unstable fixed points of ν_x = N, $3\nu_x$ = N, and $\nu_x + 2\nu_y$ = N. We find that in the region where the residual vertical chromaticity is less than + 5 and greater than - 5, stability region of ± 30 mm at small coupling ratio is assured by keeping the periodicity higher than six. The important point is that the periodicities of two, three, and four reduce the stability because the systematic resonances, $\nu_x = 3\times17 = 51$ and $\nu_x = 2\times2\times13 = 52$, are near the operation point ($\nu_x = 51.22$). In FIG.4, we show the stability limit due to the resonance of $\nu_x = 52$ for the positive chromaticity modulation. This analytical result reveals that the horizontal stability limit for the four fold symmetry case shown in FIG.3(A) is mainly due to the single first order resonance.

5. Emittance growth due to resonance crossing

We can estimate the emittance modulation due to the coupling resonance, $\nu_z \pm m\nu_s$ = N, $z = x, y$ by using the analytical method in Ref. 5. But, in our case, m is a very large number as we described in the previous paragraph and the emittance modulation is negligibly small. We don't need to take care of this emittance growth in installing RF sextupoles in the SPring-8 storage ring.

6. Conclusion

In order to modulate the chromaticity up to ~ 5 with the RF sextupoles in the SPring-8 storage ring keeping the stability of a single particle, we need six sextupoles periodically installed. And also, the possible chromaticity modulation is within ± 2.5 to keep the lifetime of 10 hours.

References

[1] T. Nakamura, Proc. Particle Accelerator Conference and International Conference on High-Energy Accelerators, Dallas, Texas U.S.A., May 1- 5 (1995).

[2] Y. Kamiya, OHO'84, pp II49 - II52 (1984) in Japanese.

[3] M. Sands, Stanford Linear Accelerator Center, SLAC-121, pp 141 - 147 (1970).

[4] R. Nagaoka et al, Nucl. Instr. and Meth. **A302**, 9-26 (1991).

[5] T. Suzuki, Particle Accelerators **18**, 115- 128 (1985).