Diffraction Limited Resolution of a Synchrotron Radiation Beam Profile Monitor

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1. Introduction

The transverse profile of an electron beam measured by imaging visible synchrotron radiation is broadened by chromatic aberrations, monochromatic aberrations, and diffraction. We can avoid chromatic aberrations by observing monochromatic synchrotron radiation. For monochromatic aberrations such as spherical aberrations and comatic aberrations, generally they are sufficiently small if we use aplanatic combination lenses. Therefore, the resolution of a synchrotron radiation beam profile monitor is dominated by diffraction.

In this report, the theory of diffraction and the imaging of synchrotron radiation are reviewed in section 2. In section 3, the theory is applied to a synchrotron radiation beam profile monitor to be installed at the machine diagnostic beamline BL38B2, and the diffraction limited resolution is estimated by numerical calculation.

2. Diffraction and Imaging of Synchrotron Radiation

We assume a beam profile monitor model wherein the synchrotron radiation is diffracted by several slits before being imaged by an ideal thin lens (Fig. 1). The slits are part of the accelerator vacuum chamber and light transport line which intercepts the light. The diffraction limited resolution of the profile monitor is evaluated by calculating an image of a single electron moving along the ideal orbit.

The monochromatic electric field of synchrotron radiation on the first slit plane is calculated by Fourier transforming the radiation field derived

$$\vec{E}_{\omega}^{(1)}(x, y) = \frac{e}{4\pi\varepsilon_0} \int \frac{\vec{n}(t') \times \left[\{\vec{n}(t') - \vec{\beta}(t')\} \times \vec{\beta}(t') \right]}{cR(t') \left[1 - \vec{n}(t') \cdot \vec{\beta}(t') \right]^3} \exp(-i\omega t) dt$$
(1)

$$\vec{n}(t') = \frac{\vec{R}(t')}{R(t')}$$
(2)

$$t = t' + \frac{R(t')}{c}, \qquad (3)$$

where $\vec{\beta}(t')$ is the velocity of an electron divided by light velocity c, and $\vec{R}(t')$ is the coordinate of the observation point viewed from the position of the electron.

The light is diffracted by the first slit and the distribution on the second slit plane at a distance L2 is given by the Rayleigh-Sommerfeld diffraction f

$$\vec{E}_{\omega}^{(2)}(x, y) = \frac{i}{\lambda} \int \frac{L_2}{R^2} \vec{E}_{\omega}^{(1)}(x', y') \exp\left[-i\frac{2\pi R}{\lambda}\right] dx' dy'$$

$$R = \sqrt{(x - x')^2 + (y - y')^2 + L_2^2},$$
(5)

where λ is the wavelength of light which is assumed to be much smaller than R. A twodimensional integral is evaluated in the aperture of the first slit. When light is diffracted by several slits, the light distribution on each slit plane is calculated by applying the formula successively.

The light is imaged by an ideal thin lens. The lens transforms the phase of light as,

$$\vec{E}_{\omega}^{(L)}(x, y) = \vec{E}_{\omega}^{(N)}(x, y) \exp\left[i\frac{2\pi}{\lambda}f(x, y)\right]$$

$$f(x, y) = \sqrt{x^{2} + y^{2} + L_{0}^{2}} + \sqrt{x^{2} + y^{2} + L_{I}^{2}} - L_{0} - L_{I}.$$
(7)

The physical interpretation of the phase shift is a transformation of the spherical wave diverging from a point at a distance Lo in front of the lens to the spherical wave converging toward a point at the distance L1 behind the lens.

The electric field on the image plane is calculated by Rayleigh-Sommerfeld formulation,

$$\vec{E}_{\omega}^{(i)}(x, y) = \frac{i}{\lambda} \int \frac{L_{I}}{R^{2}} \vec{E}_{\omega}^{(L)}(x', y') \exp\left[-i\frac{2\pi R}{\lambda}\right] dx' dy'$$
(8)
$$R = \sqrt{(x - x')^{2} + (y - y')^{2} + L_{I}^{2}} . (9)$$
(9)

The image of single electron $I\omega(x,y)$ which is observed by detectors such as a CCD camera is,

$$\mathbf{I}_{\mathbf{\omega}}(\mathbf{x}, \mathbf{y}) = \left| \vec{\mathbf{E}}_{\mathbf{\omega}}^{(1)}(\mathbf{x}, \mathbf{y}) \right|^{2} .$$
(10)



Figure 1 Model of a synchrotron radiation beam profile monitor

3. Resolution of the Beam Profile Monitor at BL38B2

3-1) Model of the BL38B2 Beam Profile Monitor

The beamline BL38B2 is a dedicated bending magnet beamline for accelerator beam diagnostics. A synchrotron radiation beam profile monitor is to be developed at BL38B2.

A simple model of the beam profile monitor is assumed as shown in Fig. 2. The two slits represent the crotch absorber and the X-ray mask, respectively.



Figure 2 Model of the synchrotron radiation beam

profile monitor to be developed at BL38B2.

An X-ray mask is introduced to absorb radiation power, which concentrates on the median plane, so that the light extraction mirror downstream of the mask is not deformed by thermal stress due to radiation power. The magnification factor of the lens is assumed to be one, and the distance LI from the lens to the image plane is equal to the distance Lo from the source point to the lens. The diameter of the lens is assumed to be 80 mm.

3-2) Electric Field on the Crotch Absorber

The trajectory of an electron in the bending magnet was calculated by assuming a magnetic field distribution obtained by measurement of a prototype magnet. The distribution of the electric field on the crotch absorber plane was calculated according to equation 1 at the wavelength $\lambda = 500$ nm.





Figure 3 Distribution of the horizontal component of electric field Ex, ω on the crotch absorber plane. The wavelength is 500 nm.

Figure 3a shows the horizontal (x) distribution of the horizontal component of electric field Ex,ω . The amplitude of the electric field is squared and converted to the corresponding photon flux. The flux is not uniform, because the light source point is near the edge of the bending magnet and the radiation from the fringing field contributes to the intensity. Figure 3b shows the vertical (y) distribution of the horizontal component of electric field Ex,ω .

The vertical (y) distribution of the vertical component of electric field Ex, ω is shown in Fig. 4. On the median plane y = 0, the amplitude vanishes and the phase has a jump of π radians.



Figure 4 Distribution of the vertical component of electric field Ey, ω on the crotch absorber plane. The wavelength is 500 nm.

3-3) Synchrotron Radiation Image of a Single Electron

The diffraction limited resolution or the image of a single electron $I\omega(x,y)$ was calculated at a wavelength of 500 nm (Fig. 5). In the figure, the image by horizontal component Ex,ω has three peaks. This is caused by diffraction by the X-ray mask, which forms a double slit along the vertical direction. The horizontal full width of the image is ± 0.2 mm, and the vertical full width is ± 0.4 mm, when the full span of three peaks is considered.

The main peak of the image by vertical component Ey,w is split. This is because the phase of Ey,w changes by an amount of radians at the median plane (y = 0) while the amplitude of Ey,w is vertically symmetric (Fig. 4). Therefore, the Rayleigh-Sommerfeld diffraction integral vanishes on the median plane. The vertical component of the electric field is not ordinarily used in beam profile

a) Image of single electron by horizontal component



b) Image of single electron by vertical component



Figure 5 Diffraction limited resolution of the synchrotron radiation beam profile monitor to be developed at BL38B2. a) and b) show images by the horizontal and vertical components of synchrotron radiation at a wavelength of 500 nm, respectively.