Model Calibration via Measurement of Betatron Functions and Phase Advances

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1. Introduction

Linear optics measurement is an essential issue for parameter tuning at a storage ring, but betatron functions and phases cannot be obtained consistently through the conventional measurements. Therefore, we are developing a new method to estimate betatron functions and phases consistently on the basis of an orbit response matrix and a ring model.

2. Theory

The orbit response matrix in this case is defined by the changes of orbits at the beam position monitors (BPMs) generated by kicks using steering magnets. This matrix, named M_{meas}, can be obtained experimentally. On the other hand, for given magnet gradients in accelerator optics modeling programs, the response matrix, named M_{mod}, can be calculated. The idea behind the model calibration method [1, 2] is that parameters in the model like quadrupole strengths and steering magnet strengths are optimized until the deviation between the model response matrix (M_{mod}) and the measured one (M_{meas}) becomes minimum. The objective function for fitting is written by

$$\chi^2 = \sum_{i,j} \frac{(M_{mod,ij} - M_{meas,ij})^2}{\sigma_i^2} \equiv \sum_{i,j} V_{k(ij)}^2 \quad (1)$$

where, for the SPring-8 storage ring, the sum is over the 568 (285 horizontal and 283 vertical) steering magnets and the 576 (288 horizontal and 288 vertical) orbit data at the BPMs . The $\{\sigma_i, i = 1, \cdots, 576\}$ are the measured rms noise levels for the BPMs.

It should be noted that minimizing $\| V \|^2$ (the norm of vector $\vec{V}$) is equivalent to minimizing χ^2. Starting with a set of initial values, q_{n0}, for the model parameters, such as the quadrupole gradients, the initial vcetor $\vec{V_0}$ and its first-order derivatives with respect to the model parameters can be calculated. The χ^2 is minimized by solving the following equation assuming V_k is a linear function of q_n

$$V_{k0} = -\sum_n \frac{\partial V_k}{\partial q_n} \triangle q_n \quad (2)$$

where the changes of the model parameters, $\{\triangle q_n, n = 1, 2, \cdots\}$, are the closest solution of Eq. (2) which can be solved in a straight forward manner by the least squares method. Since the orbit response matrix does not depend linearly on the quadrupole graidents, an iteration process will continue until the solution converges to the best set of parameters q_n^*.

The parameters varied to fit the data are: (1) the quadrupole gradient errors in the quadrupole magnets (480), (2) the quadrupole field errors in the sextupole magnets (336) due to beam offsets in the sextupoles, (3) the calibration factors of the BPMs (576), (4) the calibration factors of the steering magnets (568), and (5) the energy shifts associated with changing the strengths of the horizontal steering magnets (285). When a horizontal steering magnet strength is changed in a dispersive section, the energy shift is expressed by the following equation [3]

$$\frac{\triangle E_i}{E} = \frac{\theta_{x,i}\eta_{x,i}}{\alpha L_0} \quad (3)$$

where $\theta_{x,i}$ is the change of the i-th horizontal steering magnet strength, $\eta_{x,i}$ is the dispersion at the steering magnet, α is the momentum compaction factor, and L_0 is the circumference of the ring. This energy shift results in orbit shifts in the dispersive sections. Because there is no direct way to measure the dispersion at the steering magnets, the $\frac{\triangle E_i}{E}$ values are taken as fitting parameters. Using the measured dispersion at the BPMs, it is possible to fit the energy shifts associated with each horizontal steering magnet.

The total of 2245 parameters should be optimized to fit the 163584 elements in the SPring-8 ring response matrix. This means that at least a memory area of $3GB$ is required for computer simulation, this memory area size is beyond the capability of our computer system. Accordingly, some resonable constraints should be introduced to the system to be solved.

3. Simplified Model of Ring

Although model calibration methods have been successfully used to analyze a small storage ring, such as ALS [4], NSLS VUV and X-ray [2] rings, it is very difficult to apply such methods to a large ring, such as the SPring-8 storage ring. The following two factors are crucial elements limiting the capability of the methods. Firstly, the ring is very large with many BPMs, steering magnets, quadrupoles, and sextuploes, so the coefficient matrix becomes huge. Therefore, a memory area of $3GB$ is required as explained in the above section.

Secondly, the ring has large natural chromaticities and is very sensitive to sextupoles, so it is impossible to store a beam and measure the orbit response matrix with the sexupoles turned off. With the sextupoles turned on, the number of model parameters to fit the measured response matrix is increased and the numerical accuracy is deteriorated. In addition, since the sextupoles are always close to the quadrupoles in the ring, the method itself cannot distinguish the quadrupole field errors coming from quadrupoles and comeing from sextupoles by using the orbit data with a few μm monitoring errors. Here, the fitted model parameters suffer large systematic errors. This means that it is meaningless to calculate the quadrupole field errors from magnets one by one. Instead only the effective integrated quadrupole field errors in the girders are important in simulations of such a type.

Therefore, we take two steps to simplify the model for the ring: (1) The steering magnets in one quarter of the ring are used (72 horizontal, 72 vertical). (2) The effective integrated quadrupole field errors in the girders are regarded as fit parameters. The ring has 48 cells with three girders in each cell. Since the change of the phase advance in the middle girder is double that in the two outside girders, we assign two effective integrated field errors in the midlle girder to reduce systematic errors. Consequently, we have four effective integrated quadruploe field errors in each cell. The total number is 192.

By using this simplified model of the ring, only 984 parameters should be varied to fit the 41472 elements of the response matrix. The coefficient matrix is largely reduced. As a result, the computer memory needed is reduced from $3GB$ to $330MB$ which is acceptable for our computer system.

4. Results

The accuracy of the model calibration depends on both random and systematic errors. In order to reduce the effect of random errors, such as the random noise of the BPMs, we should use the changes of steering magnets as much as possible to increase the signal to noise ratio. However, in order to reduce the effect of systematic errors, such as the nonlinearity of BPMs and the nonlinearity of higher order fields, we should keep the changes of steering magnets as small as possible. Accordingly, the magnitude of changes of steering magnets of $50\mu rad$, is chosen to produce approximately $0.6mm$ rms changes in the orbits at the BPMs to measure the response matrix.

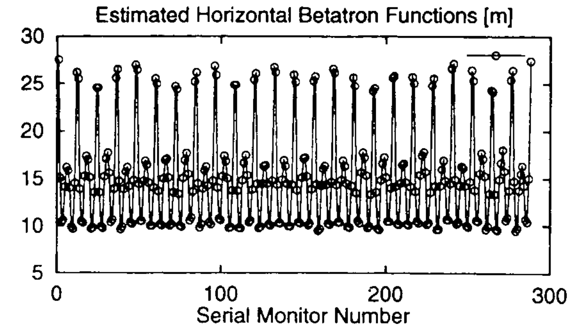

Fig. 1 Estimated horizontal betatron function

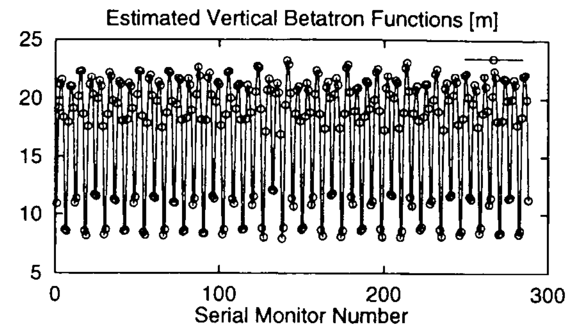

Fig. 2 Estimated vertical betatron function

The agreement between the model and measured horizontal resolution and vertical orbit shifts are $3.8\mu m$ rms and $4.3\mu m$ rms, respectively. Hence, the calculated response matrices almost converge down to the horizontal and vertical resolution of BPMs of $2.9\mu m$ rms and $3.5\mu m$ rms, respectively. The remaining difference probably comes from the simplification of the ring model. Although the accuracy of the method is difficult to determine, the fitted betatron functions and phases have relative errors less than 1% and 0.05% rms, respectively. In addition, the calibration factors of BPMs and steering magnets, and the effective integrated focusing error distributions in the girders are obtained consistently. The estimated betatron functions are shown in Fig. 1 (horizontal) and Fig. 2 (vertical). The rms modulation of the betatron functions is about 4% at both the horizontal and vertical planes.

Table. 1 Designed, measured, and fitted tunes

Tunes	Designed	Measured	Estimated
Q_x	51.3081	51.234± 0.001	51.2354
Q_y	16.4217	16.307± 0.001	16.3077

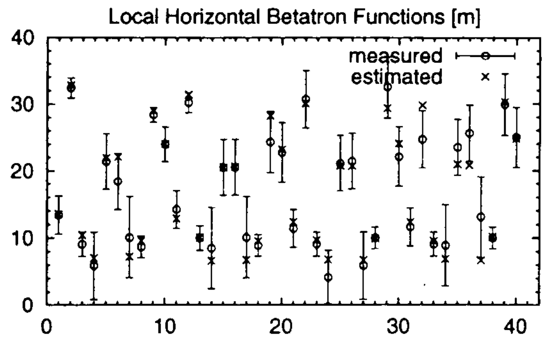

Fig. 3: Local horizontal betatron functions at the serial quadrupoles of four straight cells (i.e., cell numbers 6, 18, 30, 42). Since each cell has 10 quadruploes, the total number is 40. This means 1 to 10 for cell 6, 11 to 20 for cell 18, 21 to 30 for cell 30, 31 to 40 for cell 42.

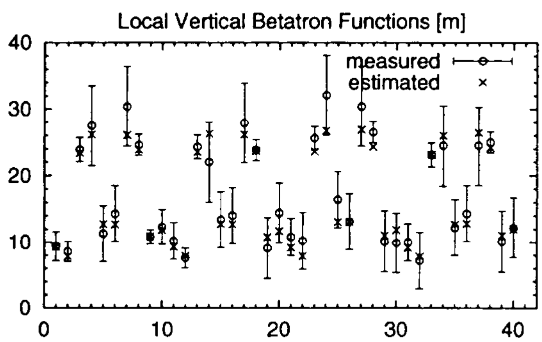

Fig. 4: Local vertical betatron functions at the serial quadrupoles of four straight cells (i.e., cells 6, 18, 30, 42). See Fig. 3 for details.

The optimized model parameters also show a good agreement with other optics measurements. Table 1 shows a comparison between measured tunes and estimated tunes. Figures 3-4 and Figures 5-6 show a comparison of local β-functions and averaged β-functions between measured and calculated. The measured local β-functions can only become available at the quadrupole magnet locations in four straight cells (i.e. cells 6, 18, 30, 42). The averaged β-functions were measured at the ten families of the quadrupole magnets. In both measurements, the β-functions were estimated by measuring the tune shifts through the following equation $\triangle\nu = \frac{\triangle K \cdot L}{4\pi}\beta$.

5. Conclusion

The model calibration method is a powerful way of calibrating the linear optics of a storage ring consistently. In order to calibrate the SPring-8 storage ring, the concept of effective integrated quadrupole field errors is introduced. As a result, the β-functions predicted by the model agree with measured β-functions. The leakages of local bump orbits, by

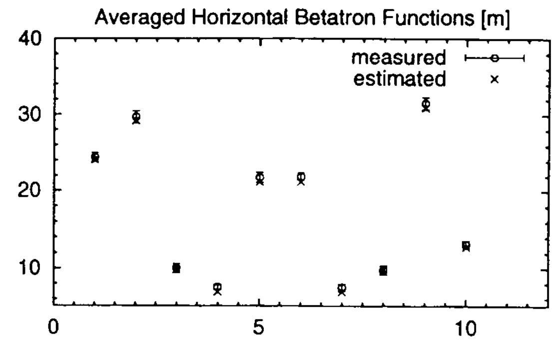

Fig. 5: Horizontal averaged betatron functions for the serial quadrupole families of the ring. As each family of quadrupole magnets is powered serially, we cannot measure the betatron functions one by one. Accordingly, only the averaged betatron function for each family was measured. Since each cell has 10 quadrupoles, the total family number is 10.

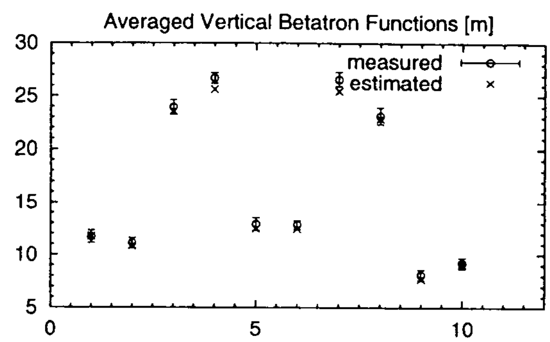

Fig. 6: Vertical averaged betatron functions for the serial quadrupole families of the ring. See Fig. 5 for details.

which we can check both the local betatron function and phase values, are being measured to further verify the validity of the model calibration method.

References

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