Perturbative Formulation of the Nonlinear Dispersion

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1. Introduction

Recently, for the stored beam in the electron storage ring, there has been increasing interest in making the bunch length short. One of the important essences for the short bunch is the smallness of the momentum compaction factor. In order to make a momentum compaction factor extremely low, one should control not only the linear part of the dispersion function but also the nonlinear part. Hence the development of analytical expression for the nonlinear dispersion is significant to give a scheme of precisely controlling a momentum compaction factor. Up to the second order the perturbation of the nonlinear dispersion is studied by second-order transfer matrix method [1] and by simplified differential equations of motion [2]. The latter method is completed by J.-P. Delahaye and J. Jäger [3] who gave the formal expression of the nonlinear dispersion up to the second order. We then accomplish the closed form of the recurrent equations for higher order terms of the dispersion function by means of the Hamiltonian formalism and solve the equations perturbatively up to the fourth order. The detail of the present report can be found in [4].

2. Formulation

Starting from the Hamiltonian with a momentum deviation δ from the design value, the equation of motion for a particle with off-momentum is written down, whose periodic solution is the dispersion function. Here we assume that there is no vertical bending field, so that we have no vertical dispersion. Expanding the horizontal motion with respect to the momentum deviation δ, we derive the recurrent equations for higher order terms of the nonlinear dispersion function, which have the following general expression:

$$\eta_n'' + \left(K_x^2 + g_0\right)\eta_n = \Omega_n\left(\eta_0, \cdots, \eta_{n-1}\right). \quad (1)$$

Here we expand the transverse motion as

$$x = \eta_0\delta + \eta_1\delta^2 + \eta_2\delta^3 + \eta_3\delta^4 + \eta_4\delta^5 + \mathcal{O}(6), \quad (2)$$

and the inhomogeneous term $\Omega_n\left(\eta_0, \cdots, \eta_{n-1}\right)$ is derived perturbatively, e.g.

$$\Omega_0 = K_x,$$
$$\Omega_1 = g_0\eta_0 - \frac{1}{2}\lambda_0\eta_0^2 - K_x\left(1 - \frac{1}{2}\eta_0'^2\right) + 2K_x^2\eta_0 - K_x^3\eta_0^2,$$

and so on. Here K_x is the curvature of the bending magnet, and g_0 and λ_0 the strength of the quadrupole and sextupole magnets, respectively. Since the recurrent equations have the same form as harmonic oscillation with the driving force, the formal solutions of the equations can be easily obtained in terms of the Green function [5]. One then finds that the highest pole of the magnetic field appearing in the explicit expressions of the nonlinear dispersion corresponds to the order of the expansion. For example, the sextupole magnet first appears at the first order, the octupole at the second order, and so on. This fact suggests that in principle we can independently adjust each order term of the nonlinear dispersion by using magnets with suitable multipole field.

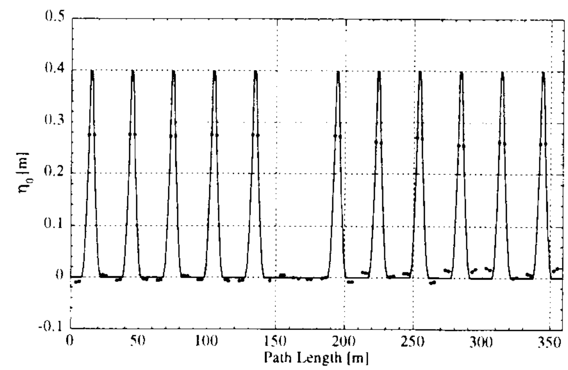

Fig. 1: Linear dispersion function at a quarter of the storage ring. The solid line denotes the numerical calculation and the full circle the measured values.

3. Numerical Study

The numerical integration can be carried out by the transfer matrix method. The driving terms of the equations can be regarded as the curvature of a sector magnet and hence the periodic solutions of the harmonic equations are easily derived from the transfer matrix. For the storage ring of SPring-8, which is the brilliant light source facility with electron beam energy 8 GeV, the nonlinear dispersion functions for the design optics are numerically calculated up to the fourth order. We show the zeroth and the first order of the dispersion function in Figs. 1 and 2. As the actual orbit is not know precisely, we use the design optics to calculate the nonlinear dispersion. Although for the second and the third order nonlinear dispersions can be calculated as well, but we do not display the figures for the sake of saving space. To clarify the condition where we can use the design optics for the calculation of the nonlinear dispersion, we calculate it for the cases with some realistic closed orbit

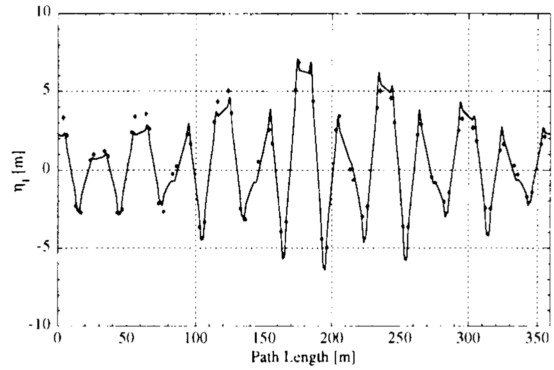

Fig. 2: The first order nonlinear dispersion function at a quarter of the storage ring. The solid line denotes the numerical calculation and the full circle the measured values.

distortion (COD). The result implies that, if the distortion of the linear dispersion is small, up to the third order there is little difference between the ideal dispersion calculated from the design optics and the dispersion of the actual optics with some COD's. On the other hand, one finds that at the fourth order the difference grows larger. It is then concluded that up to the third order we can discuss the nonlinear dispersion by means of our formulation with the design optics under the condition that the leakage of the linear dispersion is made to several percent order compared to the peak value.

4. Experimental Study

The measurement of the nonlinear dispersion was carried out at the storage ring, where the COD is corrected less than one hundred μm in r.m.s. value and hence the leakage of linear dispersion is less than several percent of the peak value. The full circles in Figs. 1 and 2 indicate the measured values of the dispersion function. We found that up to the second order there are no remarkable discrepancy between the theory and the experiment. On the contrary to the numerical calculation, we do not see a good agreement between the theoretical and the measured values at the third order of the dispersion. This can be explained by the narrow available momentum range to fit the dispersion. At the range, the contribution of the fourth power of the momentum deviation to the dispersion, i.e. the third order term becomes comparable to the order of the BPM noise level, several μm.

5. Discussions

We derived the recurrent expressions for the higher order terms of the nonlinear dispersion function for a ring with a large radius of curvature. The measurement of the nonlinear dispersion at SPring-8 agrees fairly well with the formula up to the second order. One of the reasons of this good agreement is that since the COD is well corrected, the estimation of the nonlinear dispersion by using the design optics approximates to the one of the actual orbit. On the other hand, the reason of the larger disagreement at the third order and the higher orders is that the range of the momentum deviation over which one fits the nonlinear dispersion is limited by the momentum acceptance: although the larger momentum deviation is necessary for estimating the higher order terms of the nonlinear dispersion, only the limited region of the momentum is available. In the restricted region the difference of the orbit due to the higher order terms is small, so that it is difficult to derive the higher order terms by means of the polynomial fitting.

In conclusion, it is possible to estimate the nonlinear dispersion function up to the second order by using the present formulation. Although there remains an ambiguity to use the third order term of the dispersion calculated from the design optics, the following fact may support the smallness of the deviation from the design value due to the COD. By means of the nonlinear dispersion, we can predict where the electron beam is lost when the RF power is abruptly turned off. At the SPring-8 storage ring, the design nonlinear dispersion predicts the beam to be lost at the place just after missing-bend straight sections. This is indeed observed in machine operation of SPring-8: the radiation level at the missing-bend straight section increases when the beam is aborted by suddenly turning off the RF power.

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