Method to Control Coupling Ratio in SPring-8 Storage Ring

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1. Introduction

In order to achieve low vertical emittance, we need to suppress (a) energy transfer from a horizontal betatron oscillation to a vertical one through skew quadrupole fields and (b) spurious vertical dispersion. As a first step toward extremely low vertical emittance and also to study Toucheck effect on beam lifetime, we developed a new method to control the coupling ratio of two eigen modes on the basis of eigen mode analysis.

In usual operation of the SPring-8 Storage Ring, vertical emittance is extremely small, about 10 pm•rad (~ 0.2 % by the ratio of vertical emittance with horizontal one) because of the precise alignment of main magnets. This fact is the key to think about a method for the control as described in the section 3.

By our method, we can precisely control the coupling ratio of the Storage Ring in a wide range at least from 0.2 to 100 %. It is also applicable to the correction of coupling between horizontal and vertical betatron oscillations.

2. Transformation to Eigen Mode

Time evolution of a transverse linear motion at a certain observation point is described by 4 × 4 one turn matrix M_{xy}.

$$M_{xy} \equiv \begin{pmatrix} \overrightarrow{M_{11}(s)} & \overrightarrow{M_{12}(s)} \\ \overrightarrow{M_{21}(s)} & \overrightarrow{M_{22}(s)} \end{pmatrix} , \qquad (1)$$

where $\overrightarrow{M_{ij}(s)}$ represents a 2 × 2 partial matrix. Provided that a storage ring has ideal condition without any error, the horizontal and vertical phase spaces, (x, x') and (y, y') comprise eigen spaces. In this case, the off-diagonal partial matrices in Eq. (1) are zero. When skew quadrupole fields are once introduced in the ring, horizontal and vertical oscillations are mixed and they are no longer eigen modes. This means that the off-diagonal partial matrices become non zero. By using a linear transformation [1], we can move from x-y coordinate to u-v coordinate where new eigen spaces, (u, u') and (v, v') can be defined. The one turn matrix in this coordinate takes the form of

$$M_{uv} \equiv \begin{pmatrix} \vec{U}(s) & \vec{0} \\ \vec{0} & \vec{V}(s) \end{pmatrix} . \qquad (2)$$

We can define Twiss parameters in the u-v coordinate as well and express (u, u') and (v, v') in the same form as (x, x') and (y, y') by using the Twiss parameters as

$$w(s) = \sqrt{\varepsilon_w \cdot \beta_w(s)} \cdot cos(\varphi_w(s) + \varphi_{w0}), \qquad (3)$$
$$w'(s) = -\sqrt{\varepsilon_w \cdot \gamma_w(s)} \cdot sin(\varphi_w(s) + \varphi_{w0}),$$

$$\varphi_w(s) = \int_{s0}^{s0+s} ds_1 \frac{1}{\beta_w(s_1)} , \quad \gamma_w(s) = \frac{1 + \alpha_w^2}{\beta_w(s)} ,$$

$$\alpha_w = -\frac{1}{2} \cdot \frac{d\beta_w(s)}{ds} , \quad w = u \text{ or } v .$$

where εw and βw are respectively the emittance and betatron function of the w plane. The one turn matrix M_{uv} can be built from the partial matrices of the M_{xy} by the equations below.

$$\vec{U}(s) = \overrightarrow{M_{11}(s)} - \frac{\det(\vec{R})}{\mu} \vec{R}^{-1} \cdot \overrightarrow{M_{21}(s)}, \qquad (4)$$

$$\vec{V}(s) = \overrightarrow{M_{22}(s)} + \frac{1}{\mu} \vec{R} \cdot \overrightarrow{M_{12}(s)} , \qquad (5)$$

$$\vec{R} = -\frac{\overrightarrow{M_{21}(s)} + S \cdot \overrightarrow{M_{12}(s)}^{T} \cdot S^{T}}{\pm\mu \cdot \sqrt{\chi} \cdot \mathrm{tr}(\overrightarrow{M_{11}(s)} - \overrightarrow{M_{22}(s)})} ,$$

$$\chi = \frac{2 \cdot \det(\overrightarrow{M_{21}(s)}) + \mathrm{tr}(\overrightarrow{M_{12}(s)} \cdot \overrightarrow{M_{21}(s)})}{\left[\frac{1}{2} \cdot \mathrm{tr}(\overrightarrow{M_{11}(s)} - \overrightarrow{M_{22}(s)}) \right]^2} ,$$

$$\mu = \pm\sqrt{\frac{1}{2} \cdot \left(1 \pm \frac{1}{\sqrt{\chi}} \right)} , \quad S \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

In the above equations, the superscript T represents the transpose of a matrix. By using betatron and dispersion functions in the u-v coordinate, the emittance of two eigen modes can be calculated with the well known formula for natural emittance [2].

3. Method to Control Coupling Ratio

Here, we emphasize again that the nominal coupling ratio is negligibly small in the Storage Ring, ~ 0.15 %. Our method stands on this fact and comprises two parts. One is to construct a model to describe a real ring. The model defined here has the same dependence of eigen tunes on optics as a real ring. The other is to calculate emittance of each eigen mode for the constructed model by using lattice functions in an eigen coordinate. The model construction depends on strength of the coupling as discussed below.

Weekly coupled regime

In the case where the coupling ratio is small, that is, the resonance excitation is week, we can analyze the coupled oscillation by perturbative treatment based on the single resonance approximation. This means that the x-y coordinate is approximately regarded as an eigen coordinate. The vertical emittance induced by the resonance is thus determined by the single global

parameter, called the resonance excitation term, not by local parameters. Accordingly, the model we need to estimate the coupling ratio is one which reproduces the dependence of measured tunes on the optics with simple distribution of skew quadrupole fields.

Strongly coupled regime

In the case where the coupling ratio is large, that is, the resonance excitation is strong, we can not neglect the distribution of error sources. Since the natural coupling ratio of the Storage Ring is small as mentioned above, we can add known skew fields to adjust the coupling ratio. If these fields are much stronger than unknown error fields distributed in the Storage Ring, we can approximate the real ring as the ring having only known skew fields added.

From the small to large coupling ratio, we can control its value with high precision by using our method. This is a very powerful tool especially in the machine study where we need to adjust the coupling ratio to an arbitrary value precisely.

4. Application Example

Experimental results only for the weekly coupled regime are discussed here. We changed the coupling ratio in the Storage Ring from ~ 0.2 to 20 % by changing the operation point. This was carried out by adjusting two families of quadrupole magnets in the dispersion-free sections.

The measured and calculated eigen tunes are shown in Fig. 1 as a function of the quadrupole strength. We calculated the eigen tunes here with the ring having only the single local bump of 3 mm as an error source. This amplitude was obtained by fitting the measured eigen tunes shown in Fig. 1. The calculated values have a good agreement with the measured ones.

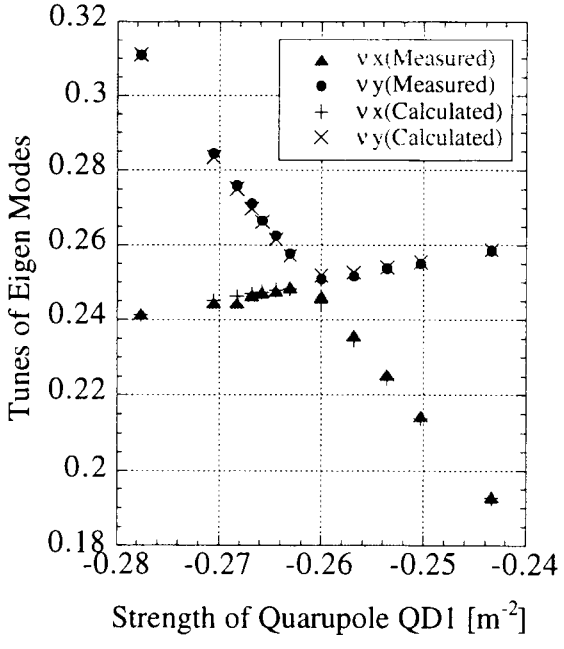

Fig. 1 Comparison between measured and calculated eigen tunes in the Storage Ring.

The difference between measured eigen tune and calculated one by our model is ~ 0.008 as an r.m.s. value.

In Fig. 2, the coupling ratio is shown at each operation point, which were estimated by both the perturbation theory and the eigen mode analysis. The discrepancy between two kinds of data in Fig. 2 becomes larger at large coupling ratios. This is

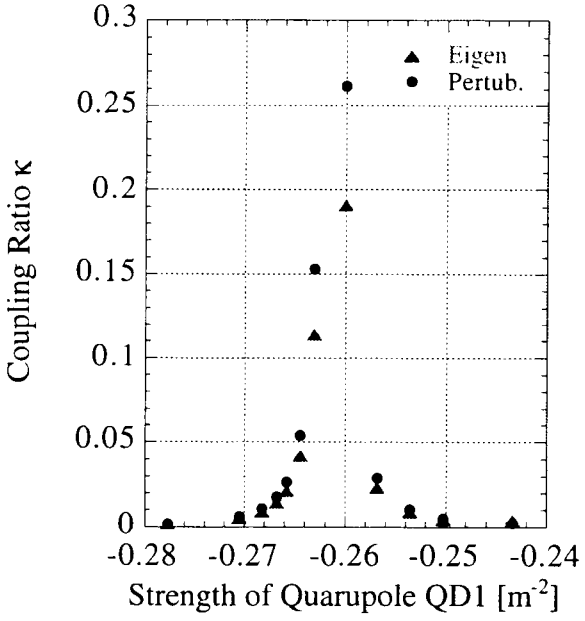

Fig. 2 Comparison between coupling ratios calculated by the perturbation theory (filled circles) and our method (filled triangles) based on the eigen mode analysis.

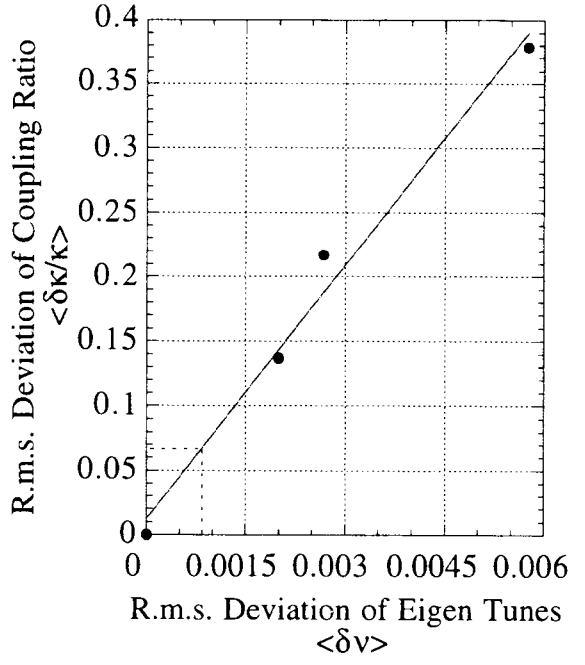

Fig. 3 R.m.s deviation of coupling ratio <δκ/κ> versus r.m.s. deviation of eigen tunes <δν>. Ten operation points were used for averaging.

because the simple perturbation theory is only valid in a weekly coupled regime. The eigen mode analysis without any perturbative treatment is free of this limit.

To roughly estimate errors of the calculated coupling ratios, we investigated variation of both coupling ratios and eigen tunes by changing a single bump amplitude by 5 %, 12.5 %, and 25 %. The dependence of coupling ratio on eigen tune deviation for this model case gives a crude scale for the calculation error. The results are shown in Fig. 3. The calculation error depends almost linearly on the eigen tune deviation. Figure 3 shows that an r.m.s. calculation error of the coupling ratio becomes an order of several percent when an r.m.s. eigen tune deviation is about 0.008, i.e. the case in Fig. 1.

References

[1] H. Koiso; OHO'91, p.I-1 - p.I-22 (1991) in Japanese, K. Ohmi; OHO'91, p.II-1 - p.II-33 (1991) in Japanese.

[2] M. Sands; Report SLAC-121 (1970).