Focusing Property by a Bent Crystal for High Energy Synchrotron Radiation

Hitoshi YAMAOKA and Tetsuya ISHIKAWA

The Institute of Physical and Chemical Research (RIKEN), Mikazuki, Sayo, Hyogo 679-5143, Japan

The resolution of a bent crystal monochromator is determined by the natural width of the crystal, crystal perfection, deviation of the crystal curvature from the ideal one, and distortion due to a heat load or other reasons. Here, we show a simple analysis for a doubly-bent crystal to determine the optimum focusing conditions for high energy synchrotron radiation geometrically[1].

Two coordinate systems, (x, y, z) and (X, Y, Z), are introduced, as shown in Fig. 1(a). The wave vector of incident x-rays, $\boldsymbol{k}$, is expressed in (x, y, z) coordinates as $\boldsymbol{k} = \boldsymbol{k}(\cos\sigma_z\sin\sigma_x, \cos\sigma_z\cos\sigma_x, \sin\sigma_z)$. The reciprocal lattice vector, $\boldsymbol{h}$, is written in (X, Y, Z) coordinates as $\boldsymbol{h} = (-h_X, -h_Y, h_Z)$. Taking $\tan\xi = -h_X/h_Z$, $\tan\eta = -h_Y/h_Z$ and $h^2 = h_X{}^2+h_Y{}^2+h_Z{}^2 = h_Z{}^2(\tan^2\xi+\tan^2\eta+1)$, we may write $\boldsymbol{h} = \{h/(\tan^2\xi+\tan^2\eta+1)^{1/2}\}(-\tan\xi, -\tan\eta, 1)$, where $h = |\boldsymbol{h}|$. The relation of these coordinates is expressed as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \qquad (1)$$

where θ is the incident angle of the x-rays.

The reciprocal lattice vector is expressed in (x, y. z) coordinates as $\boldsymbol{h} = [h/(\tan^2\xi+\tan^2\eta+1)^{1/2}](-\tan\xi, -\tan\eta\cos\theta-\sin\theta, -\tan\eta\sin\theta+\cos\theta)$. From the Bragg relation, $\boldsymbol{k}_h = \boldsymbol{k}+\boldsymbol{h}$ and $|\boldsymbol{k}| = |\boldsymbol{k}_h|$, and the relation of a two-coordinate system, the angle deviation, δθ, from the Bragg angle of a doubly-bent crystal at point A in Fig. 1 is written as

$$\delta\theta = \theta_{inc}-\theta_B = \arcsin(-\boldsymbol{k}\bullet\boldsymbol{h}/kh) - \theta_B$$
$$= \arcsin\{[1/(\tan^2\xi+\tan^2\eta+1)^{1/2}][-\tan\xi\cos\sigma_z\sin\sigma_x - \cos\sigma_z\cos\sigma_x(\tan\eta\cos\theta+\sin\theta) + \sin\sigma_z(-\tan\eta\sin\theta+\cos\theta)]\} - \theta_B. \qquad (2)$$

where θ is the incident angle of the x-rays. When $\theta_{inc} = \theta_B$, eq. (1) is written approximately as $2\xi\sigma_x-2\eta\cos\theta-(\xi^2+\sigma_x{}^2+\eta^2)\sin\theta = 0$. Normally we can derive the angles of ξ and η from the approximate relation, $\xi = p\sigma_x/N$ and $\eta = p\sigma_z/(R\sin\theta)$, where p, N and R are the distance from the source to crystal, the sagittal radius and the meridian radius, respectively. In eq. (2) if we set $\sigma_x = 0$, $\xi = 0$ and $\theta = \theta_B$, the angle deviation becomes $\delta\theta = \eta-\sigma_z$. When $\delta\theta = 0$, $\eta = \sigma_z$. Then we can get the relations of $p=R\sin\theta$ and $q/p = 1$ for the symmetrical reflection, where q is the distance from the crystal to the focus point. If $\eta = \sigma_z$, $\theta = \theta_B$ and $\delta\theta = 0$ in eq. (2), the following relation is approximately derived, neglecting higher-order terms, $2\xi\sigma_x-(\xi^2+\sigma_x{}^2)\sin\theta = 0$. Then we find $N = (1-\cos\theta)p/\sin\theta$. It should be noted that at a small incident angle, the above result agrees with the relation $N' = p\sin\theta/2$ derived from the known relation of $N' = \{2pq\sin\theta/(p+q)\}$ when $p/q = 1/3$. In practice the ratio of $\{(N-N')/N\}$ is less than 1% for an incident angle less than 10°. The optimum focus point of the sagittal focus which minimizes the energy spread, is apparently different from that of the meridian focus.

Another possible angle deviation is caused by the reflection at point C in Fig. 1(b). We take into account the depth effect along the Z direction in Fig. 1 for the incident x-rays of $\sigma_z = 0$ that land on a location with a different $\boldsymbol{h}$ and a different Z. In Fig. 1(b), η' is written as $\eta' = N(1-\cos\xi)/(R\tan\theta)$. To change N, there is an optimum for the magnification at about 0.2. To change R the deviation monotonously decreases as a function of the magnification. It should be noted that if we choose the focusing condition so that $q/p = 1/3$ in the sagittal direction and $q/p = 1$ in the meridian direction, the angle deviation at the point C gives a wider energy spread than that at point B.

Reference

[1] H. Yamaoka, K. Ohtomo and T. Ishikawa, to be published in J. Synchrotron Rad. (1998) 5

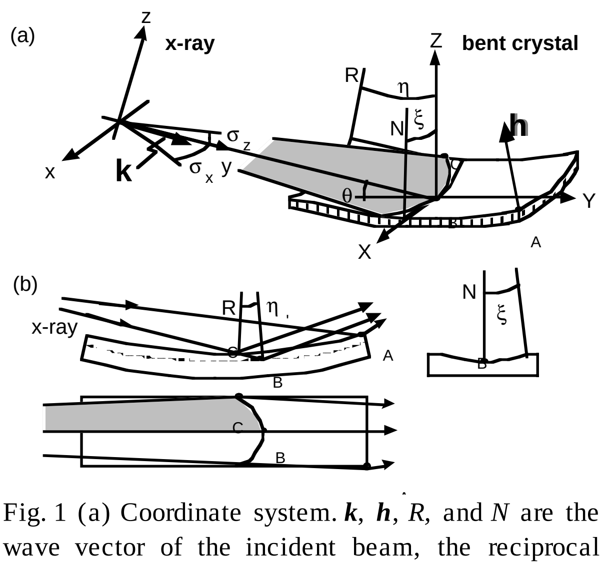

Fig. 1 (a) Coordinate system. $\boldsymbol{k}$, $\boldsymbol{h}$, R, and N are the wave vector of the incident beam, the reciprocal lattice vector of the crystal, the meridional radius, and the sagittal radius, respectively. (b) Another point C on the crystal surface that affects the angle deviation.