# Energy Spread Measurement of the SPring-8 Storage Ring with Chromatic Sideband Peak Height of Betatron Oscillation Spectrum

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## 1. Introduction

The energy spread of electron beams in storage rings are usually estimated from bunch length, the transverse beam size at finite dispersion and spectrum of light generated by transverse optical klystron which is a special type of insertion devices.

We propose a new method to measure the energy spread of the beam, using the chromatic sideband peaks in the betatron oscillation spectrum. The preliminary measurement of the energy spread of the electron beam in the SPring-8 storage ring was performed.

## 2. Method

If we measure the frequency response of the betatron motion of a bunch in a storage ring which has finite chromaticity, we see several sideband peaks which apart from the main peak of betatron frequency with the distance n  $f_s$ , where n =0, ± 1, ± 2,... and  $f_s$  is the synchrotron frequency. The typical data of the frequency response of the vertical betatron motion of the SPring-8 storage ring is shown in Fig. 1.

#### **Frequency Response of Betatron Motion**

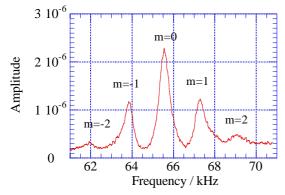


Fig. 1. Typical data of the frequency response of the betatron motion of the beam in the SPring-8 storage ring. The direction of betatron motion is vertical and the vertical chromaticity is 8.8 (y~1) and the height of the m=0 peak is reduced to be a half of that at chromaticity 0 as shown in Fig. 2. The mode number m of each peak is also shown. This signal was averaged during 10 sweeps.

From Ref. [1], if we assume the RF potential is harmonic, the height of these sideband peaks depend

only on the value  $y = \xi \sigma_{\delta} / v_s$  where  $\xi$  is the chromaticity,  $\sigma_{\delta}$  is the energy spread,  $v_{\beta}$  and  $v_s$  is the betatron tune and synchrotron tune, respectively, and the dependence on y is shown in Fig. 2 and Fig. 3 which shows the ratio of the peak height of the sideband to the peak height of the main peak (m=0).

In this paper, the definition of the chromaticity  $\xi$  is  $\Delta v_{\beta} = \xi$  ( $\Delta E/E$ ) and is different from the Ref. [1] where chromaticity is a normalized value;  $\xi / v_{\beta}$ .

From the data of the height of sideband peaks, we have the values of y and we can obtain  $v_s$  from the distance of sideband peaks each other or synchrotron frequency measurement, we have  $\xi \sigma_{\delta}$ .

The chromaticity  $\xi$  is measured by the dependence of betatron frequency on energy shift as mentioned later, then we have  $\sigma_{\delta}$ 

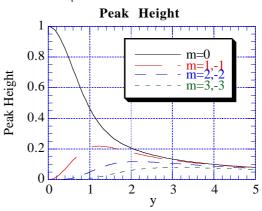


Fig. 2. Calculated relative peak height for m=0,  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$  based on Ref.[1]. y =  $\xi \sigma_{\delta} / v_s$ 

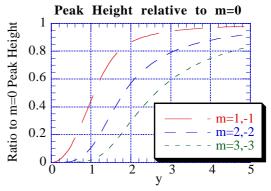


Fig. 3. Calculated ratio of peak height of  $m=\pm 1,\pm 2,\pm 3$  to the amplitude of the peak of m=0. y  $= \xi \sigma_8 / v_s$ .

## 3. Experiment

The setup of the measurement is shown in Fig. 4 and is usually used as a tune monitoring system.

The chromaticity was varied by changing the strength of sextupole magnets of the ring and measured by taking the dependence of the betatron tune on the RF frequency using the relation

 $\Delta v_{\beta} = (\xi / \eta) (\Delta f_{RF} / f_{RF})$ 

assuming the momentum compaction factor  $\eta$  was  $1.46 \times 10^{-4}.$ 

Spectrum Analyzer

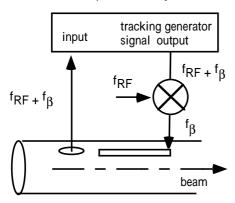


Fig. 4. Setup for measurement of frequency response of betatron motion of the beam in the storage ring. A spectrum analyzer generated the signal of frequency  $f=f_{RF} + f_{\beta}$ , where  $f_{RF}$  is RF frequency and  $f_{\beta}$  is the betatron frequency, and this signal was down-converted by a double-balanced mixer to the frequency  $f_{\beta}$  to shake the beam. The transverse motion of the beam excited by this signal is detected by a button-type pick-up electrode at frequency  $f=f_{RF} + f_{\beta}$ .

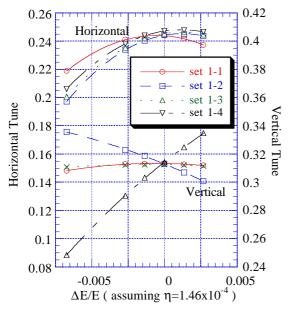


Fig. 5. Energy dependence of the betatron tune (m=0) assuming the momentum compaction factor is 1.46  $\times 10^4$ . Three lines are for different sets of sextupole magnet strength. The chromaticity is  $\xi = dv_\beta / d\delta = dv_\beta / d(\Delta E/E)$ .

At the measurement of the synchrotron frequency, the synchrotron motion was excited by adding phase modulation to RF acceleration voltage and the frequency response of the phase oscillation amplitude of the beam was measured.

The signal of the transverse motion of the beam was detected by a button-type electrode attached to the beam pipe of the ring. The center frequency of the spectrum analyzer was tuned to the betatron frequency (m=0 peak) and the span of the frequency was set to be 10kHz which is wider enough to cover sideband peaks of  $n =\pm 1, \pm 2$  at nominal value of the synchrotron frequency; ~ 2kHz.

The signal to excite the betatron motion is generated by a tracking generator of the spectrum analyzer and is fed to the strip-line. The tracking generator generates a signal of the frequency tuned to the sweeping frequency of the spectrum analyzer hence we can obtain frequency response of the betatron motion of the beam.

The dependence of the betatron frequency (m=0) on the energy shift controlled by the RF frequency is shown in Fig. 5. The chromaticity is obtained by  $\xi = dv_\beta/d$  ( $\Delta E/E$ ). We assume the momentum compaction factor  $\eta$  to be the design value;  $1.46 \times 10^{-4}$  to convert the RF frequency shift to the energy shift using the relation  $\Delta E/E = 1/\eta$  ( $\Delta f_{RF}/f_{RF}$ ) because we do not have scheme to measure absolute value of the energy shift.

#### 4. Results

The energy spread obtained from the measured peak height of m= 0,  $\pm 1$ ,  $\pm 2$  are shown Fig 6, 7 and 8. The error caused by reading error from the data recorded as video images of the spectrum analyzer display, is shown in Fig. 9. This figure shows that the error is lowest at the chromaticity 4 to 8. The measured value of the synchrotron tune  $v_s$  is 0.00778 at these measurement.

These result shows that the energy spread of the beam is  $1 \times 10^{-3}$  which is as expected from the magnetic field measurement of the dipole magnets [2].

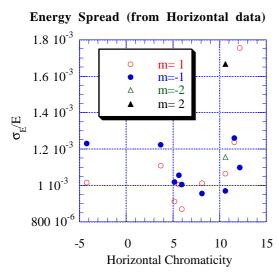


Fig. 6. The energy spread of the beam obtained from the ratio of chromatic sideband peak height relative to m=0 main peak for horizontal betatron motion. From Fig ., the error is smallest at chromaticity ~ 4-8 for m= $\pm$  1 peaks and ~8 for m= $\pm$ 2 peaks as shown in Fig. 3.

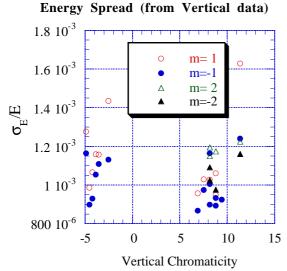


Fig. 7. The energy spread of the beam obtained from the ratio of chromatic sideband peak height relative to m=0 main peak for vertical betatron motion.

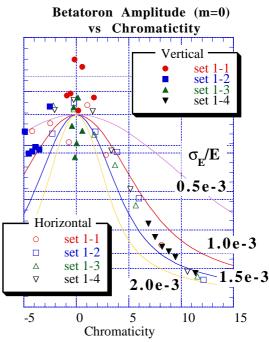


Fig. 8. Energy spread measured by the peak height of main peak m=0 for different set of sextupole field strength. The lines in the figure are expected value of each energy spread which shown in the figure. This results show the energy spread is between  $1 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ . Peak value at chromaticity =0 is the average of the measured values.

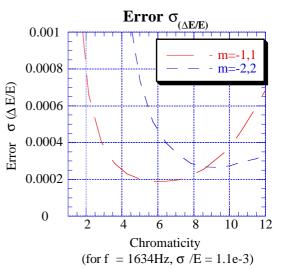


Fig. 9. Estimated error of the measured energy spread using the ratio of peak height, caused by reading error from data as video images of the spectrum analyzer display recorded as video images. This figure shows that the error is smallest at chromaticity 4 to 8.

## References

- T. Nakamura, "Excitation of Betatron Oscillation under Finite Chromaticity", SPring-8 Annual Report 1998, (1998).
- [2] K. Tsumaki, SPring-8 Annual Report 1998, (1998).